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THE

PHYSICAL REVIEW.

THE THEORY OF IONIZATION BY COLLISION.

II. Case of Inelastic Impact.

By K. T. Compton.

IN the preceding paper[1] an investigation was made of the distribution of velocities among the ions in a discharge tube when the current through the gas is sufficiently small that the potential gradient may be considered uniform. Townsend[2] has shown that under these conditions α, the average number of new ions formed per centimeter path by the impacts of each negative ion (electron), may be expressed by an equation of the form

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right), \tag{23}$$

where X is the potential gradient and p the pressure. This relation has been supported by all experiments with gases in which the impacts of electrons with gas molecules are inelastic, *i. e.*, when the electron loses its kinetic energy at each collision.

Various assumptions regarding the mechanism of ionization by collision have led to various forms of the function f. Townsend,[3] assuming that ionization always occurs at an impact when the electron has moved through a potential drop exceeding the minimum value V_0, derived the equation

$$\alpha = \nu e^{-\frac{\nu V_0}{X}}, \tag{24}$$

where ν is the average number of collisions with gas molecules made by each electron in a centimeter path. By assuming arbitrary values of the constants ν and V_0 for each gas this equation may be made to fit the experimental results, but the values of the constants so chosen are

[1] Phys. Rev., April, 1916.

[2] Ionization of Gases by Collision, p. 17.

[3] *Ibid.*, p. 22.

at wide variance with the values which have been determined directly. Recognizing this, Townsend has recently[1] suggested that V_0 in his equation be considered as the average rather than the minimum ionizing potential. Under these conditions equation (24) must be considered largely as an empirical equation.

Much more satisfactory is the relation derived by Bergen Davis[2] on the assumption that ionization occurs at an impact whenever the electron possesses a velocity whose component normal to the surface of the molecule (considered spherical) exceeds the minimum value corresponding to V_0. This equation is

$$\alpha = \nu\left[e^{-\frac{\nu V_0}{X}} + \frac{\nu V_0}{X} Ei\left(-\frac{\nu V_0}{X}\right)\right], \tag{25}$$

where $Ei(\ )$ represents the exponential integral. It will be shown that Bergen Davis' assumptions really require a modification of equation (25). Further discussion of this equation will appear later.

The purpose of this paper is to derive a new expression for α and to test it with such experimental data as are available. It is found to agree well with experimental results as regards both the variation of α with X and the values of V_0 for various gases.

Theory.—The two factors which determine the laws governing the production of ions by impact are the distribution of velocities among the colliding electrons and the function of the velocity which represents the probability that an electron will ionize at an impact. Thus if $F(V)dV$ represents the probability that an electron has fallen through a potential drop between V and $V + dV$ and if $f(V)$ represents the probability that an electron with a velocity V ionizes when it collides, then $f(V)F(V)dV$ is the probability that an electron will possess a velocity corresponding to V and will ionize when it collides. Thus

$$P = \int_0^\infty f(V)F(V)dV$$

is the probability that any electron, chosen at random, ionizes when it collides. This quantity, when multiplied by ν, is equal to α.

$F(V)dV$ is given by equation (7) of the preceding paper.[3] As to the form of $f(V)$, we know that $f(V) = 0$ when $V \leqq V_0$ and that it must approach the value unity when V is large. The simplest form of such a function is one which fits the facts as far as we know them; we shall therefore assume that

$$f(V) = \frac{V - V_0}{V} \quad \text{when} \quad V \geqq V_0. \tag{26}$$

[1] Electricity in Gases, p. 295.

[2] PHYS. REV., 24, p. 93, 1907.

[3] *Loc. cit.*

This relation is essentially the same as that which follows from Bergen Davis' assumption regarding the necessary condition for ionization at an impact. It may be correct, however, even if the interpretation given by Bergen Davis is not true. The bearing of certain experiments by Glasson[1] on this point will be discussed later.

If we make these substitutions in the equation for P we obtain

$$P = \int_{V_0}^{\infty} (1 + P) \frac{\nu}{X} \frac{V - V_0}{V} e^{-(1+P)\frac{\nu V_0}{X}} dV,$$

of which the solution is

$$P = e^{-(1+P)\frac{\nu V_0}{X}} + (1 + P) \frac{\nu V_0}{X} Ei\left(- (1 + P) \frac{\nu V_0}{X} \right),$$

where $Ei(\ )$ denotes the exponential integral. Thus we find

$$\alpha = P\nu = \nu \left[e^{-(1+P)\frac{\nu V_0}{X}} + (1 + P) \frac{\nu V_0}{X} Ei\left(- (1 + P) \frac{\nu V_0}{X} \right) \right], \quad (27)$$

which conforms to equation (23) since ν is proportional to p.

Discussion of Equation.—Equation (27) differs from the expression given by Bergen Davis by the presence of the factor $(1 + P)$ whose omission from his equation results from neglecting to consider the effect of the newly formed ions on the velocity distribution.

It is impossible to solve equation (27) for α or P explicitly. However P may be found in terms of $(1 + P)(\nu V_0/X)$ (see Làska's "Sammlung von Formeln"), from which I have computed the following table of values:

TABLE I.

P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$	P	$\frac{\nu V_0}{X}$
0.00099	5.00	0.0731	1.446	0.3893	0.2880	0.725	0.0580
0.0032	3.99	0.1485	0.872	0.518	0.1645	0.804	0.0333
0.0106	2.97	0.2008	0.666	0.574	0.1271	0.854	0.0216
0.0375	1.963	0.2762	0.4705	0.656	0.0846	1.000	0.0000

In order to test equation (27) P was plotted as a function of $\nu V_0/X$, whence P, and therefore α, could be determined graphically for any given value of $\nu V_0/X$.

Comparison with Experiment.—(*a*) *Variation of α with X.*—Numerous experiments on the variation of α with X in various gases contained between parallel plate electrodes afford a means of testing equation (27). Of the available data, those for nitrogen, carbon dioxide, hydrochloric acid and water vapor are found to be in excellent agreement with equation

[1] Phil. Mag., 22, p. 647, 1911.

(27). In helium, argon and hydrogen the collisions are entirely or partly elastic and equation (27) cannot be applied. They will be considered in a subsequent paper.

In selecting experimental data I have taken the results of the most recent, or otherwise apparently most reliable, experiments on each gas. All results which are appreciably affected by the error in such measurements pointed out by Farwell,[1] or for which there is an appreciable difference between the velocity distributions given by equations (8) and (10) of the preceding paper, have been rejected. In view of the labor of gathering and testing the available data it may be of value to include them here. The data given in Table II. have been reduced by equation (23) so as to apply to a pressure of 1 mm.

TABLE II.

p = 1 mm.

N_2[2]		CO_2[3]	
X	a	X	a
43.75	0.021	87.5	0.62
87.5	0.337	175	2.32
131	0.96	350	5.65
175	1.77	700	10.64
262	3.59	875	11.83
350	4.42	1050	12.68
525	6.41		
H_2O[4]		HCl[5]	
32.5	0.02	55.5	0.12
54	0.29	68.5	0.41
65	0.57	69	0.36
97.5	1.25	103	1.10
103.5	1.41	104	1.38
107.5	1.56	112.5	1.36
134	2.25	116	1.66
163.5	2.80	138	2.30
205	3.79	172	3.38
221	4.08	203.5	4.23
307	5.49	232	5.08
374	6.23	305	6.96
		357	8.00
		407	9.18
		535	11.20
		715	13.40

[1] PHYS. REV., 5, p. 149, 1915.

[2] H. E. Hurst, Phil. Mag., 11, p. 535, 1906.

[3] *Ibid.*

[4] J. S. Townsend, Phil. Mag., p. 389, 1903.

[5] *Ibid.*

The exactness with which equation (27) conforms to these data is indicated by Fig. 2. The solid curve in each case represents the graph of equation (27), the constants ν and V_0 being chosen to give the best agreement between the curve and the data. I find that equations (24)

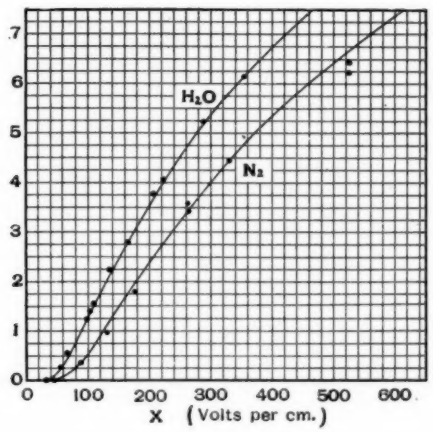

Fig. 2(*a*).

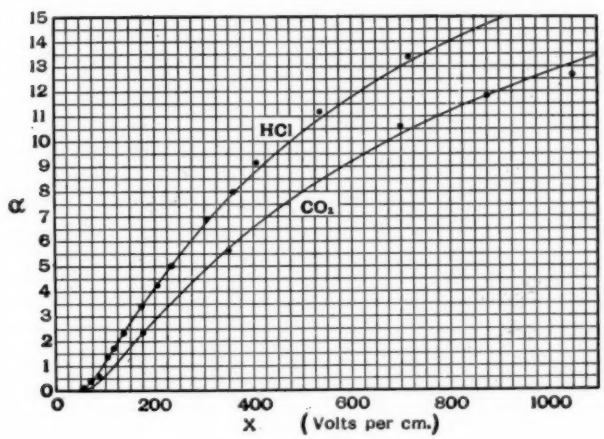

Fig. 2(*b*).

and (25), for Townsend's and Bergen Davis' theories, agree nearly, though not quite, as well as the present theory. Possibly all are within the limits of experimental error. But when the values of the constants necessary to bring this agreement are compared with the results of direct measurements there is evident support of the present theory.

Values of V_0.—The minimum ionizing potential V_0 for a few gases has

been measured directly by Franck and Hertz[1] and by Pawlow.[2] Their values are given in Table III. in the column marked *Exp.* In their

TABLE III.

Values of V_0.

	Exp.	Exp.*	Equation (27).	Townsend.	Davis.
N_2	7.5	10.0	10.05	27.5	12.36
CO_2			8.2	23.3	9.63
H_2O			7.47	22.4	8.98
HCl			6.05	16.5	7.30
He	20.5	22.5	22.5	12.3	5.6

experiments the source of electrons was a hot filament, from which the electrons escape with a definite velocity distribution. Pawlow determined this distribution and found that the fastest electrons which he could detect escaped with an initial velocity corresponding to about 2.5 volts. Now as the accelerating potential is raised ionization will first be noticed when these fastest electrons acquire the minimum ionizing velocity. Apparently, however, this initial velocity has not been included in the published values of V_0. It appears to me, therefore, that these values should each be increased by about 2.5 volts. These revised values are placed in the column marked *Exp**. In the remaining columns are given the values of V_0 which cause the curves representing the three theories to accord best with the data in Table II. I have also anticipated the results of the following paper and included the value of V_0 for the elastic gas helium, since the theoretical treatment is exactly the same as in the present paper except for the effect of elastic impact. The results for hydrogen have not been included since in this case impact appears to be partly but not wholly elastic and I have not been able to solve the equation for such a case. Townsend's and Davis' equations do not really apply to the case of elastic impact, and the value for helium calculated from these equations has been included in order to illustrate the important fact that when applied to cases of elastic impact they tend to give values of V_0 which are too small. Since for nitrogen they give values of V_0 which are too large it is not likely that the impacts in nitrogen are elastic enough to make the present treatment appreciably in error.

Where comparison with direct experiment is possible the present theory is seen to fit the facts most acceptably. In fact, if the suggested correction of the experimental data is true, the agreement is almost exact. It is unfortunate that comparison is possible in so few cases. It is known

[1] Verh. d. D. Phys. Ges., 16, p. 457, 1914. [2] Roy. Soc. Proc., 90, p. 398, 1914.

however, that CO_2, H_2O and HCl may be ionized with comparative ease and to this extent there is support of the theory.

Another possible test of these values of V_0 is afforded by photoelectric experiments in which the minimum ionizing potential for a gas may be calculated from the maximum wave-length of light which can ionize the gas. These wave-lengths, however, are so extremely short that no accurate data are available and it can only be said that no existing evidence of this sort casts doubt on the values of V_0 given by the theory.

Bergen Davis[1] has recently modified his equation to apply to the experimental arrangements employed by Franck and Hertz and has found good support of his equation in their experimental data. It is to be noted that their measurements were carried on under conditions in which the factor $(1 + P)$ which appears in equation (27) does not need to be considered, whence the formulæ in Bergen Davis' and the present theories become identical. Unfortunately the lack of a source of electrons of homogeneous velocities complicates the interpretation of such results.

Values of ν.—Another test of the relative merits of the theories is afforded by a comparison of the calculated values of ν with those determined experimentally. From the assumptions underlying the theories ν, for inelastic impact, should equal $\pi R^2 N$, where N is the number of molecules per unit volume and πR^2 is that part of the molecular cross section which is effective in stopping the impinging electrons. In Table IV. I have included the values of ν calculated from the three theories and also calculated on the assumption that R is the known molecular radius.[2]

TABLE IV.

Values of ν *for* $p = 1$ *mm.*

Gas.	$\pi R^2 N$	Equation (27).	Townsend.	Davis.
N_2	34.5	22.5	12.4	17.6
CO_2	49.5	30.0	20.0	23.2
H_2O	47.0	21.7	12.9	15.7
HCl	46.0	33.3	22.2	26.8
He	13.5	10.0	2.8	3.05

Evidently the part of the molecular cross section which is effective in stopping the electron is less than that which functions in collisions of molecules with each other. One cannot speak very definitely about this point since the meaning of a collision is not entirely clear in either case.

[1] PHYS. REV., p. 118, 1915.

[2] Kaye and Laby's Tables.

An interesting hypothesis is this: that the explanation of the above results is to be found in the discrete structure of the molecule, for it is conceivable that a rapidly moving electron might penetrate a molecule without colliding with any of its constituent parts. If this explanation is correct, the ratio of the actual values of ν to the values of $\pi R^2 N$ in Table IV. would represent the probability that an electron could not pass through the space occupied by a molecule without colliding with some of its constituent parts.

Glasson has found[1] that, with very high velocities, the number of ionizing collisions per unit path by an electron *decreases* as the velocity increases, which is contrary to what we might expect. In line with the suggestion just made, it is likely that Glasson's results are not due to a decrease with velocity in the probability of ionization at a collision, but rather to an actual decrease in the number of collisions. In other words, we may consider ν to be a function of the velocity such that ν decreases at very high velocities. Direct evidence of this is afforded by the photographs of paths of β particles through gases. The reason appears to be that a molecule has a discrete structure and that an electron has the best chance of getting through freely when its velocity is large compared with the velocities of the constituent parts of the molecule in their orbits. Such a view removes an apparent objection to equation (26).

However this may be, no test of the ionization theories by considerations of ν can be made at this time owing to the lack of experimental measurements of ν. Such measurements are now being attempted in this laboratory.

Further support of the assumptions underlying equation (27) will appear in a following paper on the subject of elastic impact.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

[1] *Loc. cit.*

THE THEORY OF IONIZATION BY COLLISION.

III. CASE OF ELASTIC IMPACT.

BY K. T. COMPTON.

Introduction.—In the preceding paper[1] there was derived an equation by which α, the average number of new ions of either sign formed per centimeter path in a gas by each electron, could be accurately calculated from a knowledge of the potential gradient X, the minimum ionizing potential V_0 and the average number of collisions ν made by each electron in a centimeter path in the direction of X. This equation was shown to accord with experimental results more accurately than any equation hitherto proposed. It was based on the assumption of inelastic impact between electrons and gas molecules and therefore does not apply to such gases as helium, argon and neon in which the collisions have been shown to be elastic,[2] *i. e.*, the kinetic energy of the colliding electron is retained except at impacts which result in ionization. The problem of deriving a theoretical relation between X, α and ν applicable to these gases is much more difficult than in the case of inelastic impact and has, so far as I am aware, been attempted only by Franck and Hertz[3] who did not obtain an equation to which experimental data could be applied.

In the present paper I have succeeded in deriving, for cases of elastic impact, an equation based on the same assumptions as those underlying the theory for inelastic gases developed in the preceding papers. This equation seems to receive very satisfactory experimental support.

Functional Relation between α, X and p.—Townsend[4] has shown that a relation of the form

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$$

must hold in all cases of ionization of gases by inelastic impact. His argument does not necessarily apply in case the collisions are elastic. It is necessary, therefore, to see whether the same type of relation is true in the case of elastic impact.

[1] PHYS. REV., this issue.

[2] Franck and Hertz, Verh. d. D. Phys. Ges., 15, p. 373, 1913; 15, p. 613, 1913; 16, p. 457, 1914.

[3] Verh. d. D. Phys. Ges., 16, p. 12, 1914.

[4] Ionization of Gases by Collision, p. 18.

Let ν represent, as formerly, the average number of impacts by each electron while advancing one centimeter in the direction of the electric intensity X. Let ν_1 represent the average number of impacts by each electron in a centimeter path in any direction. ν_1 is the reciprocal of the mean free path of the electron.

In inelastic gases ν equals ν_1, since there is no component of velocity perpendicular to the electric intensity except that arising from thermal agitation, which is negligible in comparison with the velocity due to the electric field. In the elastic gases, however, there is generally a sidewise component; the electron will bound and rebound many times and in all directions. Thus in the present case ν exceeds ν_1 by an amount which we shall now calculate.

During a free path l the electron experiences an acceleration equal to Xe/m in the direction of the electric field. If $\bar{v}$ is the average velocity of an electron just before it ionizes, $\bar{v}/2$ is its average velocity and hence $2l/\bar{v}$ is the mean free time. Therefore

$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{Xe}{m}\left(\frac{2l}{\bar{v}}\right)^2$$

gives the average distance moved in the direction of the electric field during one free path. The reciprocal of s is ν, whence

$$\nu = \frac{1}{s} = \frac{m\bar{v}^2}{2Xel^2} = \frac{m\bar{v}^2\nu_1^2}{2Xe}.$$

But Xe is the energy acquired by the electron in moving one centimeter in the direction of the field and $\frac{1}{2}m\bar{v}^2$ is the average energy of the electron when it ionizes. The ratio of these quantities evidently equals α, the number of ionizing collisions made per centimeter by each electron. Thus

$$\nu = \frac{\nu_1^2}{\alpha}, \quad \text{or} \quad \alpha = \frac{\nu_1^2}{\nu}. \tag{28}$$

But $\alpha = P\nu$, where P is the probability that an electron ionizes when it collides. Thus

$$P = \left(\frac{\nu_1}{\nu}\right)^2, \quad \text{or} \quad \nu = \frac{\nu_1}{\sqrt{P}}. \tag{29}$$

Since P is always less than unity, ν always exceeds ν_1. In the limit, with so large a field that ionization would occur at every impact, ν_1 would equal ν and P would equal unity, as is otherwise obvious. This shows that no numerical factor has been neglected in equation (29) owing to the use in the above argument of the average velocity rather than the exact velocity distribution.

Now ν_1 is proportional to the pressure p. Also it is obvious that if X varies as p, then ν varies as p. Thus, from equation (28), if X varies as p then α varies as p; whence

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p}\right). \tag{30}$$

We see, therefore, that the type of functional relation between α, X and p is the same for elastic and inelastic gases; the form of the function ψ is of course different in the two cases.

We shall proceed to determine the form of the function ψ for an elastic gas. It will be convenient to deal with ν instead of ν_1. At the end of the argument equation (29) will enable us to replace ν by the more familiar quantity ν_1.

Determination of $\psi(X/p)$.—As in the case of inelastic impact the probability that any electron, chosen at random, ionizes at an impact with a gas molecule is

$$P = \int_0^\infty f(V)F(V)dV, \tag{31}$$

where $f(V)$ is the probability that an electron with a velocity due to a potential drop V ionizes when it collides and $F(V)dV$ is the probability that an electron possesses a velocity due to a potential drop between V and $V + dV$. For the reasons given in the preceding paper we shall take

$$f(V) = 0 \qquad \text{if} \quad V \leqq V_0,$$
$$f(V) = \frac{V - V_0}{V} \quad \text{if} \quad V \geqq V_0. \tag{32}$$

It remains to determine $F(V)dV$. We shall suppose that the mean free path l and the minimum ionizing path x_0 are small in comparison with the distance between the electrodes of the discharge tube, and in testing the equation so derived we shall reject experimental data taken under circumstances in which this restriction can be shown to lead to an appreciable error. Under these conditions we may take P and ν to be uniform throughout the discharge tube.

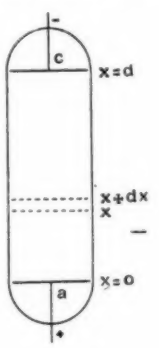

Fig. 3.

Consider the subsequent history of the electrons which start from rest in any layer dx in the gas. (Fig. 3.) Only at an ionizing collision is an electron stopped, whence two electrons start from rest as the result of each ionizing collision in dx. Thus the number of electrons starting from

rest in dx per second is $2P\nu n dx$, where n is the number of electrons entering the layer dx per second. The number of new negative ions originating per second in dx is

$$dn = -P\nu n dx,$$

whence

$$n = n_a e^{-P\nu x}. \tag{33}$$

where n_a is the number per second which reach the anode a. Therefore

$$n_0 = 2P\nu n_a e^{-P\nu x}\, dx \tag{34}$$

electrons start from rest in dx each second.

If x_0 is the distance which an electron must go in the direction of the electric force in order to acquire the minimum ionizing velocity corresponding to V_0, it is evident that none of the electrons starting in dx can ionize (or be stopped) in going a distance less than x_0. Thus

$$[F(x)dx = 2P\nu e^{-P\nu x}\, dx]_{x=0}^{x=x_0} \tag{35}$$

applies within the limits noted. In terms of potential drop this is

$$\left[F(V)dV = \frac{2P\nu}{X} e^{-\frac{P\nu}{X}V}\, dV \right]_{V=0}^{V=V_0}. \tag{35}$$

Equation (35) gives the distribution of velocities of the electrons passing any plane in the gas, for velocities less than the minimum ionizing velocity. For all of these $f(V) = 0$, whence they contribute nothing toward the value of P in equation (31).

Consider now the history of those n_0 electrons after they have gone a distance x_0. We shall call $f(x)$ the probability that one of these ionizes at an impact after having gone a distance x from its starting point in dx. $f(x)$ is obviously identical with $f(V)$ and is equal to $(x - x_0)/x$.

Let n_x be the number of the original n_0 electrons which have not been stopped by ionizations in going the distance x from dx. Of these, $\nu n_x dx'$ will collide in the ensuing layer dx'. Thus $f(x)\nu n_x dx'$ of the original n_0 electrons are stopped by ionization in the layer dx' distant x from dx. In other words

$$dn_x = -f(x)\nu n_x dx',$$

whence

$$n_x = n_0 e^{-\int_{x_0}^{x} f(x)\nu dx'} = n_0 e^{-\nu \int_{x_0}^{x} \frac{x-x_0}{x} dx'}$$

$$= n_0 \left(\frac{x}{x_0}\right)^{\nu x_0} e^{-\nu(x-x_0)}.$$

Thus

$$\frac{n_x}{n_0} = \left(\frac{x}{x_0}\right)^{\nu x_0} e^{-\nu(x-x_0)}$$

is the probability that an electron, originating in dx, goes a distance x, $(x \geqq x_0)$, without being stopped.

From this quantity and equation (35) we obtain

$$\left[F(x)dx = 2P\nu \left(\frac{x}{x_0}\right)^{\nu x_0} e^{\nu x_0} e^{-\nu(1+P)x} dx \right]_{x=x_0}^{x=\infty}, \tag{36}$$

or

$$\left[F(V)dV = \frac{2P\nu}{X} \left(\frac{V}{V_0}\right)^{\frac{\nu V_0}{X}} e^{\frac{\nu V_0}{X}} e^{-(1+P)\frac{\nu V}{X}} dV \right]_{V=V_0}^{V=\infty}, \tag{36}$$

for the velocity distribution, in terms of potential drop, of those electrons which are able to ionize while passing through any plane of the gas.

Finally, substituting from equations (32) and (36) in (31) we find

$$P = \int_{V_0}^{\infty} \frac{V - V_0}{V} \frac{2P\nu}{X} e^{\frac{\nu V_0}{X}} \left(\frac{V}{V_0}\right)^{\frac{\nu V_0}{X}} e^{-(1+P)\frac{\nu V}{X}} dV,$$

or, in terms of x,

$$P = \int_{x_0}^{\infty} 2P\nu e^{\nu x_0} \frac{x - x_0}{x} \left(\frac{x}{x_0}\right)^{\nu x_0} e^{-(1+P)\nu x} dx,$$

which is easily reduced to

$$\frac{x_0^{\nu x_0}}{2\nu e^{\nu x_0}} = \int_{x_0}^{\infty} x^{\nu x_0} e^{-(1+P)\nu x} dx - x_0 \int_{x_0}^{\infty} x^{\nu x_0 - 1} e^{-(1+P)\nu x} dx.$$

In order to plot this equation it will suffice to solve it only for integral values of νx_0, in which case the integrals are of the familiar type $\int x^m e^{ax} dx$. The factorial terms arising from the two integrals may be combined in such a way that the above equation reduces to

$$e^{P\nu x_0} = 2\nu x_0 \left\{ \frac{1}{[\nu x_0(1+P)]^2} + \frac{2(\nu x_0 - 1)}{[\nu x_0(1+P)]^3} + \frac{3(\nu x_0 - 1)(\nu x_0 - 2)}{[\nu x_0(1+P)]^4} + \cdots + \frac{(\nu x_0 - 1)(\nu x_0 - 1)!}{[\nu x_0(1+P)]^{\nu x_0}} + \frac{\nu x_0!}{[\nu x_0(1+P)]^{\nu x_0 + 1}} \right\} \tag{37}$$

This is the simplest form of the equation expressing the theory of elastic impact.

The calculation of corresponding values of νx_0 and P from this equation was exceedingly laborious. I gave νx_0 various integral values between 0 and 80 and in each case determined by trial the value of P which satisfied the equation. Some of the values so obtained are given in Table V. Since we wish to deal ultimately with the mean free path l, or its reciprocal ν_1, instead of ν, the values from the above calculations have been translated into terms of ν_1 by the use of equation (29). Instead of x_0 its more useful equivalent V_0/X has been written in Table V.

TABLE V.

$\frac{\nu V_0}{X}$	P	$\frac{\nu_1 V_0}{X}$	$\frac{\nu V_0}{X}$	P	$\frac{\nu_1 V_0}{X}$
0	1.000	0.000	10	0.0478	2.186
1	0.249	0.499	15	0.0337	2.756
2	0.161	0.802	20	0.0265	3.256
3	0.1213	1.045	25	0.0222	3.725
4	0.0984	1.255	30	0.0189	4.124
5	0.0831	1.441	40	0.0144	4.793
6	0.0723	1.613	60	0.0101	6.030
8	0.0574	1.917	80	0.0075	6.940

For any given value of $\nu_1 V_0/X$ the corresponding value of P may thus be found, whence

$$\alpha = P\nu = \nu_1 \sqrt{P} \tag{38}$$

by equation (29).

Experimental Verification of Equation (37).—Experiments on exceedingly pure helium by E. W. B. Gill and F. B. Pidduck[1] enable us to test equation (37). I have applied to their data tests similar to those mentioned in the paper on inelastic impact, whence I have rejected all values

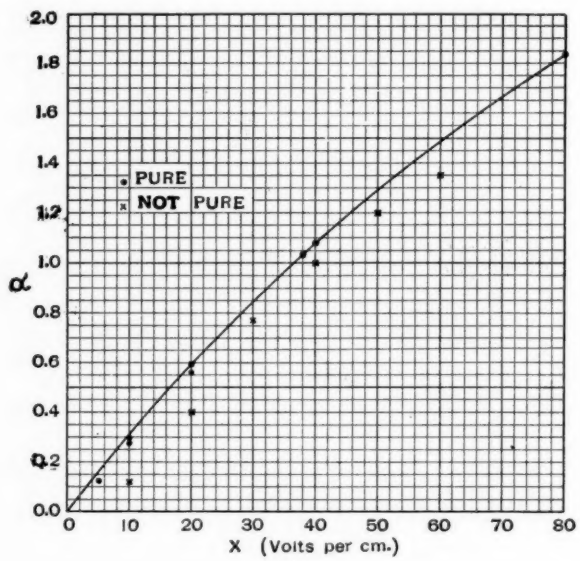

Fig. 4.
Helium.

for which P and ν cannot be considered approximately uniform throughout the ionization chamber. The values have been reduced as suggested by equation (30) to correspond to a pressure of 1 mm. The agreement

[1] Phil. Mag., 23, p. 837, 1912.

between these data and equation (37) is shown by Fig. 4 in which the dots represent the experimental data and the solid curve represents the equations (37) and (38) with the constants V_0 and ν_1 so chosen as to give the best agreement.

Additional support of the theory is found in the calculated values of V_0 and ν_1. In Table VI. the column marked *Exp.* gives the directly determined value of V_0 as published, the column marked *Exp.** gives the revised experimental value which appears to me most probable from a consideration of the experimental results[1] and the fourth column gives the value of V_0 determined by the present theory. Corresponding data from the preceding paper on inelastic impact are included for comparison.

TABLE VI.

Gas.	V_0			ν_1	
	Exp.	Exp.*	Theory.	$\pi R^2 N$	Theory.
He	20.0	22.5	22.5	13.5	10.0
A	12.0	13.5	14.5?	32.0	15.0?
N_2	7.5	10.0	10.05	34.5	22.5
CO_2			8.20	49.5	30.0
H_2O			7.47	47.0	21.7
HCl			6.05	46.0	33.3

Table VI. contains also the values of ν_1 calculated for a pressure of 1 mm. In the fifth column are given values calculated on the assumption that $\nu_1 = \pi R^2 N$, where πR^2 is the molecular cross section calculated from the kinetic theory and N is the number of molecules per unit volume. Where direct test is possible the theoretical values of V_0 appear to be accurate and the values of ν_1 appear to be consistently smaller than $\pi R^2 N$, for which a reason was previously suggested.

In the case of the elastic gas argon the ionization experiments are untrustworthy and the values calculated from them by the present theory can only be regarded as very approximate in view of the following considerations.

Effect of Impurities in the Gas.—Not only is there good agreement between theory and experiment as shown by Fig. 4, but the deviations from the theory are in a direction to be explained by traces of impurities in the gas, as is shown by the crosses which represent experiments in which the helium was known to be less pure.[2] The effect of even slight traces of ordinary gases is to give helium, in small fields, the characteristics of an inelastic gas. In other words, impurities cause the $\alpha - X$ curve

[1] See the preceding paper, *loc. cit.*

[2] E. W. B. Gill and F. B. Pidduck, Phil. Mag., 16, p. 280, 1908.

to leave the origin horizontally instead of at a definite angle. It is not surprising, therefore, that the only available data for argon do not conform accurately to the present theory when we know that the experiments were performed under conditions identical with those in which the crossed data for helium were obtained.[1] The values of V_0 and ν_1 for argon in Table VI. were obtained by constructing a curve which bore the same relation to the data for argon that the curve of Fig. 4 does to the crossed data. These values are, of course, much more uncertain than those for the other gases.

Approximate Theory of Ionization in a Mixture of Elastic and Inelastic Gases.—I have not been able to work out an exact formula to cover this case but will briefly outline a treatment which various tests have shown to be not very far from correct. The equation which we shall derive has its chief value in its ability to account for the effects of small traces of impurities in gases.

Suppose that an electron, in one centimeter path, makes an average of ν_i and ν_e collisions with molecules of inelastic and elastic gases, respectively, and let V_{oi} and V_{oe} be the minimum ionizing potentials in the two gases. By equation (21) in an earlier paper[2] we have, for the velocity distribution among the electrons approximately

$$F(V)dV = \frac{(1 + P_i)\nu_i + 2P_e\nu_e}{X} e^{-\frac{(1+P_i)\nu_i+2P_e\nu_e}{X}V} dV,$$

where P_i and P_e are the probabilities of ionization at an inelastic and an elastic impact, respectively. We have therefore, from equation (31),

$$P_i = \int_{V_{oi}}^{\infty} \frac{V - V_{oi}}{V} F(V)dV = e^{-\frac{sV_{oi}}{X}} + \frac{sV_{oi}}{X} Ei\left(-\frac{sV_{oi}}{X}\right)$$

and

$$P_e = \int_{V_{oe}}^{\infty} \frac{V - V_{oe}}{V} F(V)dV = e^{-\frac{sV_{oe}}{X}} + \frac{sV_{oe}}{X} Ei\left(-\frac{sV_{oe}}{X}\right),$$

where $s = (1 + P_i)\nu_i + 2P_e\nu_e$ and $Ei(\ )$ denotes the exponential integral.

But $\alpha = P_i\nu_i + P_e\nu_e$, whence

$$\alpha = \nu_i\left[e^{-\frac{sV_{oi}}{X}} + \frac{sV_{oi}}{X} Ei\left(-\frac{sV_{oi}}{X}\right)\right] + \nu_e\left[e^{-\frac{sV_{oe}}{X}} + \frac{sV_{oe}}{X} Ei\left(-\frac{sV_{oe}}{X}\right)\right]. \quad (39)$$

The type (elastic or inelastic) of variation of α with X depends on the relative magnitudes of the two terms in s. For example, in the case of argon or helium containing a trace of nitrogen or oxygen we can easily predict the general shape of the $\alpha - X$ curve. Since V_{oi} is considerably smaller than V_{oe}, P_e becomes negligible in comparison with P_i when X

[1] E. W. B. Gill and F. B. Pidduck, *loc. cit.*

[2] PHYS. REV., April, 1916.

is very small. Thus for small values of X the gaseous mixture shows the characteristics of its inelastic component. When X is large, however, P_e and P_i do not differ so greatly and the mixture has more nearly the characteristics of the gas which is present in the greater quantity. Though the relative effect of the inelastic constituent is always large in comparison with the proportion of this constituent present, the relative importance of this inelastic constituent diminishes as X increases. Fig. 5 illustrates this point. The upper curve refers to a pure elastic gas, the lower curve to a pure inelastic gas and the intermediate curve represents equation (39) for the elastic gas with a small proportion of the inelastic gas.

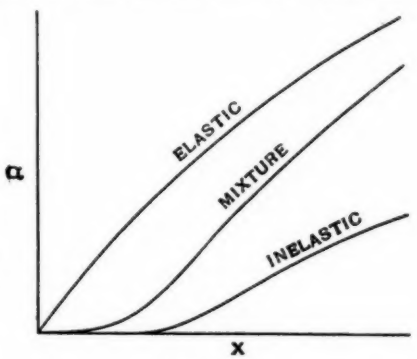

Fig. 5.

This discussion suggests the reason for the extreme electrical sensitiveness of helium and argon to the slightest traces of impurities. I have not yet succeeded in deriving an equation to replace equation (39) following the exact treatment of elastic impact given in the earlier part of this paper.

Summary.—In the first paper of this series exact expressions were obtained for the velocity distribution among the negative ions of inelastic gases and approximate expressions for this distribution in elastic gases and mixtures. These expressions applied to cases of ionization exclusively by impacts of negative ions and also to ionization by both negative and positive ions.

In the second paper an expression was derived for α, the number of new ions formed by inelastic impacts of a negative ion in a centimeter path and this expression was found to receive good experimental support.

In the present paper the functional relation between α, X and p for elastic impacts was derived and an equation for α was obtained which conformed to the best experimental data. The departure of some experimental data from this theory was shown to arise from the use of impure gases.

In all the discussion it has been assumed that velocities due to thermal agitation are negligible and that no complex ions or clusters are formed within the range of electric fields and gas pressures employed in experimental tests. The only experimental data used were those in which the effect of the finite distance between the electrodes could be neglected.

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SOME CHARACTERISTICS OF CRYSTAL DETECTORS.

BY VICTOR A. HUNT AND LAURENS E. WHITTEMORE.

THIS paper describes an investigation of the behavior of certain crystal detectors, as used in wireless telegraphy, when subjected to varying conditions of temperature, pressure and humidity.

A considerable amount of work has been done in recent years in studying the properties of various crystals and metals which have been found to be rectifiers of high frequency currents.[1] Up to the present time, however, the phenomenon of crystal rectification has been investigated chiefly by the direct application of known potential differences, both direct and alternating, to the terminals of the rectifier.

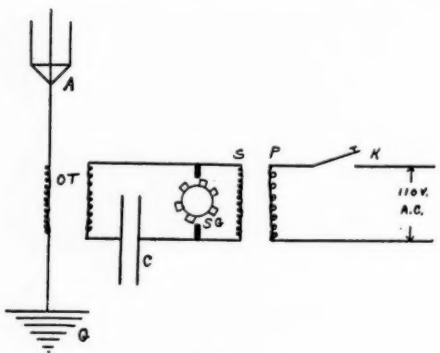


Fig. 1.

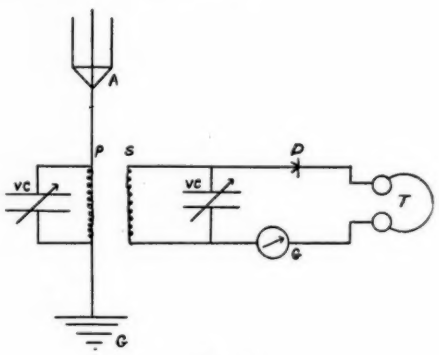


Fig. 2.

Since the most important use of crystal rectifiers at the present time is in wireless telegraphy, it was decided to undertake the investigation of the properties of four of these detectors under conditions similar to those of actual practice. Observations were made of the variations which occurred in the rectified current with changes of temperature, pressure and humidity. For the purpose we erected a sending station and a complete receiving station in different parts of the same building. Fig. 1 and Fig. 2 show simple diagrams of the two circuits. The detectors studied were galena, perikon, silicon and carborundum.

The sending station was arranged to emit a wave of about 500 meters.

[1] The following papers give a fairly comprehensive list of the recent publications on the subject of crystal rectifiers: Goddard, PHYS. REV., 34, 423, 1912; Flowers, PHYS. REV., 3, 25, 1914; Hartsough, PHYS. REV., 4, 306, 1914.

The normal current in the closed oscillatory circuit was 4.5 amperes, and the power consumed by the transformer was 240 watts. The curve *A* in Fig. 3 shows that when this current was used, such fluctuations as would be present in the power supply would produce only small variations in the galvanometer deflection at the receiving station.

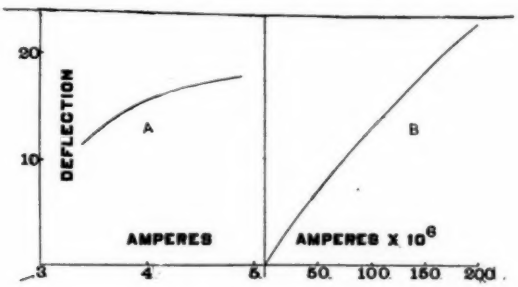


Fig. 3.

The measurements of the current in the detector were made by means of a Leeds and Northrup ballistic galvanometer of 1,069 ohms resistance and having a sensibility of 0.0026 micro-amperes per millimeter. Fig. 4

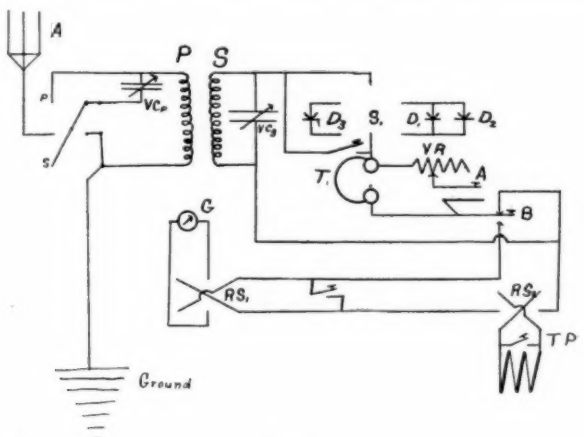


Fig. 4.

gives a complete diagram of the connection of the receiving instruments. The signals were sent out from the sending station by closing a key at the receiving station. In order to prevent any inductive effect in the galvanometer circuit from the make and break of the current in the primary of the sending transformer, the key *B* was always left open at the make and break of the sending key. The time during which the current flowed through the galvanometer was determined entirely by the time during

which the key B was closed. By means of the reversing switch RS_1 the direction of the galvanometer deflections could be changed. The switch S_1 was used to connect in an auxiliary detector with which to receive messages from other stations without disturbing the detector under test.

By far the most troublesome source of error which we encountered was an "extra" E.M.F. of variable magnitude which made its appearance continually throughout the course of the investigation. It originated at the contact of the detector and would cause a seemingly unexplainable deflection of the galvanometer even when no signals were being received. To eliminate this by inserting an opposing E.M.F. from a potentiometer in the circuit proved to be inconvenient and unsatisfactory. The method of elimination which was used consisted in the use of a thermopile which was inserted directly in series with the galvanometer, and was heated by the radiation from a thirty-two candle power carbon filament lamp. To adjust the E.M.F. generated in the thermopile the position of the lamp was varied by moving it back and forth on a sliding track. Since the "extra" E.M.F. often reversed its direction it was necessary to use a reversing switch RS_2 in order that the E.M.F. of the thermopile might always oppose it. This arrangement made it possible to eliminate entirely all extraneous currents from electro-chemical or thermo-electric sources such as have been mentioned by Flowers.[1]

The following is the method of procedure used in taking a series of readings. One observer confined his attention to the galvanometer. He first eliminated whatever "extra" E.M.F. was present by opposing it with the E.M.F. generated in the thermopile. This balance was secured by closing the key B and adjusting the distance of the lamp until the galvanometer showed no deflection. The observer in charge of the galvanometer wore the telephone receivers at all times in order to detect any errors in the readings which might occur due to the interference of atmospherics or to the sending of a nearby station, and all data which contained such errors were discarded.

The other observer then closed the key which put the sending station in operation, and, with stop watch in hand, pressed the key B for a certain length of time. The first observer then noted the maximum deflection of the galvanometer. Because of its long natural period it did not have time to come to its full deflection in the three seconds during which the current was flowing through it. The curve B, in Fig. 3, shows the deflections produced by the corresponding currents flowing for a period of three seconds.

In making a test we did not attempt to secure the most sensitive adjust-

[1] PHYS. REV., 29, 445, 1909.

ment of the detector, but tried above all to secure one which would remain permanent and which gave a clear, uniform tone. The deflections were adjusted to the range of the galvanometer scale by varying the coupling of the receiving transformer which was then kept the same throughout that particular experiment. In order to subject the detector to the various physical conditions desired, we had constructed a box which consisted of a glass battery cell with a suitable brass cover.

To heat the detector we enclosed it in the box which was then immersed in cylinder oil of a high flash point and heated by a gas burner. In order to cool the detector we suspended it in a U-shaped Dewar flask which contained liquid air. The temperature obtained was measured by means of a pentane thermometer.

The pressure in the box was varied by means of a motor driven air pump and was measured with an open tube manometer.

The humidity of the air in the box was varied by varying the concentration of a sulphuric acid solution which was placed in the bottom of the cell. The air was first dried by putting in a known volume of concentrated sulphuric acid and letting it stand for some time. To increase the humidity the acid was diluted by the addition of known quantities of water. The humidity was allowed to reach a state of equilibrium in each case. This was found to require about ten minutes. The value of the vapor pressure for each concentration was obtained from the Physikalisch-Chemische Tabellen of Landolt, Bornstein and Myerhoff.

Results.

Galena.

The first physical condition which we varied was temperature, since it was natural to suppose that changes in temperature would produce the most noticeable effect. The galena detector was carefully adjusted and placed in the closed box which was then immersed in oil and heated. The heat was supplied continuously and the galvanometer readings taken at successive intervals while the temperature was rising. The galvanometer readings were then plotted against the respective temperatures. Fig. 5 shows one of these curves and is typical of those obtained with galena. In this as well as the other figures the dotted curves indicate the results obtained with other specimens or other adjustments. The rectification falls off very rapidly with the rise in temperature, approaching zero at 170° C. So long as the adjustment remained the same there was no improvement in rectification when the detector was cooled.

The decrease in rectification was not due to increased pressure at the contact, since the needle point was suspended by a very fine coiled

spring, so that expansion due to increased temperature would not affect its adjustment. We were much gratified with the uniformity of the readings which we were able to secure with galena, since we had feared

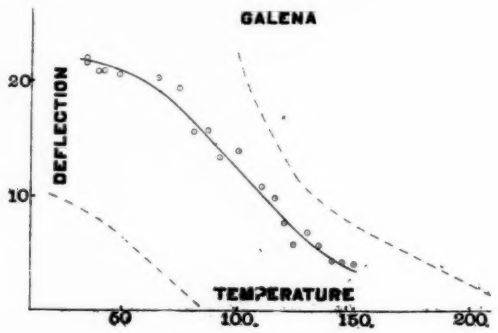


Fig. 5.

that, on account of the light contact needed, a permanent adjustment could not be preserved. The following data show the uniformity of the galvanometer readings under constant conditions at the detector.

Galvanometer Deflection for Signals Three Seconds Long.
Galena Detector under Room Conditions.

15.1
14.6
14.9
15.2
15.0
15.0
14.9
15.3
14.9
15.3
14.9
14.7
15.0

One noteworthy fact in connection with the rectification by galena is that, although in most cases the current flows from crystal to point, it was found that, with some adjustments, the current would flow in the other direction.

It is a well-known fact that when a relatively large current caused either by a heavy atmospheric discharge or by the spark at a sending station nearby, passes through a galena detector, the rectification is impaired. It has occurred to us that a probable explanation of this phenomenon is the local heating of the contact by the passage of the current producing an effect similar to the heating described above.

The temperature at which rectification becomes practically negligible is not always the same but varies with the original adjustment, being higher for better adjustments.

In cooling the galena detector to a low temperature the manipulation was not so easy, for we could not obtain the decrease in temperature as uniformly as we should have liked. By lowering the detector nearer and nearer to the surface of the liquid air in the Dewar flask we were able to obtain sufficient data to determine the behavior of the galena detector in that range of temperature. Fig. 6 shows the results of this

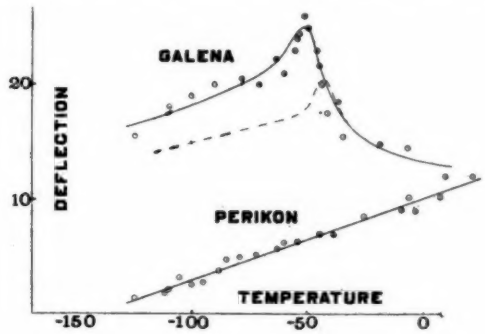


Fig. 6.

experiment. It will be noticed that at about $-50°$ C. there occurred a decided maximum of rectification, which was corroborated by successive tests.

One of the greatest difficulties which we encountered in cooling galena as well as the other detectors was the precipitation of frost upon the crystals. If any of this frost melted due to heating at the contact the effect of humidity was superimposed upon that of temperature. In addition, this moisture at the contact had a decided influence on the "extra" E.M.F. causing it to be very strong, although sometimes in one direction and sometimes in the other. The glass-hardness of the surfaces of the crystals at these low temperatures made it very difficult to retain the adjustments. It will be noted that the curve obtained by cooling continues the general trend of the one obtained by heating.

No effect on the rectification by galena could be noted when the detector was completely immersed in liquid air. Good adjustments remained good and poor ones poor.

The results of varying the humidity of the air in the closed box are shown in Fig. 7. The galvanometer deflections are shown which correspond to the vapor pressures of the various concentrations of sulphuric acid solution in the bottom of the box. It will be seen that the presence

of a very small quantity of moisture is sufficient to reduce greatly the rectification by the galena detector. Each of the points on Fig. 7 represents the average of at least ten successive readings taken after the vapor pressure had reached a state of equilibrium.

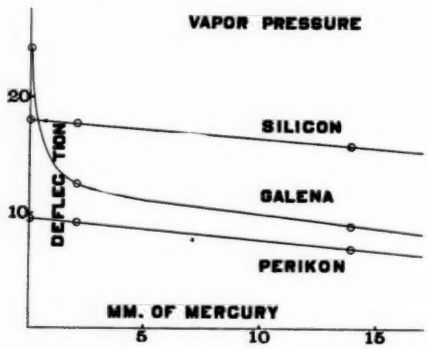


Fig. 7.

Galvanometer deflections corresponding to vapor pressures of sulphuric acid solutions.

Perikon.

The perikon detector used consisted of a contact between crystals of zincite and chalcopyrite. The effect of temperature was more marked in the case of perikon than in the case of any other detector as is shown in Fig. 8. Starting with a deflection of 3 cm. at about 55° C. for a three-

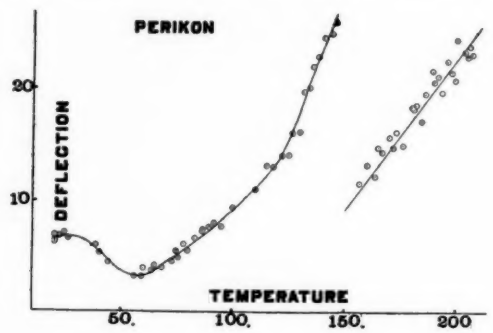


Fig. 8.

second dash, the deflection passed off the scale of the galvanometer at a temperature of about 140° C. When the length of the dash was shortened to one second the deflection became 7 cm. but rose rapidly with the temperature reaching a value of 24 cm. at 210° C.

The curve obtained upon cooling the perikon detector (Fig. 6) was perfectly continuous with the heating curve, falling rapidly with the

temperature. At no time was there any indication of a definite maximum or minimum of rectification within the temperature range which we used, from − 120° C. to 210° C. Immersion of the perikon detector in liquid air caused a complete loss of rectification.

When the vapor pressure of the space surrounding the perikon detector was changed, the results shown in Fig. 7 were obtained. The effect was not so marked as in the case of galena but there appeared a definite decrease in rectification with an increase in humidity.

Silicon.

We found the rectifying action of silicon to be very erratic. This corresponds with the statements made by other observers in regard to the properties of that substance. Miss Frances G. Wick, in a paper on the "Electric Properties of Silicon,"[1] makes the remark that "the position of the element in the periodic system between the metals and the non-metals may explain some of the deviations of its properties from those of the stronger metals."

O. E. Buckley[2] states that, "since different specimens, sometimes even if cut from the same piece, may vary widely in their electrical properties, it is important that measurements of different properties

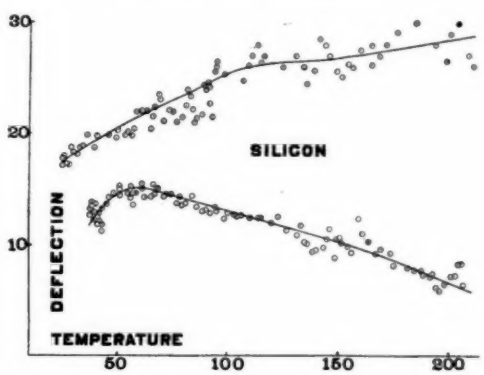


Fig. 9.

be made with the same specimen." His statement is confirmed by the results of our investigations as shown in Fig. 9. The conditions of heating were identical in obtaining both curves except that different specimens were used. In one case the rectification is seen to increase and in the other to decrease as the temperature rises.

In attempting to determine the effect of subjecting silicon detectors

[1] PHYS. REV., 25, 382, 1907.

[2] PHYS. REV., 4, 482, 1914.

to low temperatures it was found impossible to obtain any consistent results. The rectification would change very rapidly when there was apparently no cause whatsoever, and it was very hard to retain a given adjustment since the surface of the crystal became exceedingly hard and slippery at the low temperatures.

When the silicon detector was completely immersed in liquid air the sound in the telephone receivers increased in the majority of cases. It is possible that in other cases the somewhat violent boiling of the liquid air destroyed the adjustment of the light needle contact.

The behavior of silicon under changes of humidity resembles very much that of the perikon detector, the rectification decreasing somewhat with the increase of vapor pressure. These results are shown in Fig. 7.

Carborundum.

In order to secure a good adjustment with carborundum we found it necessary to use considerable pressure upon the contact point, consequently imbedding the point of the steel needle below the surface of the crystal. The adjustment was very permanent and could not easily be impaired by vibration.

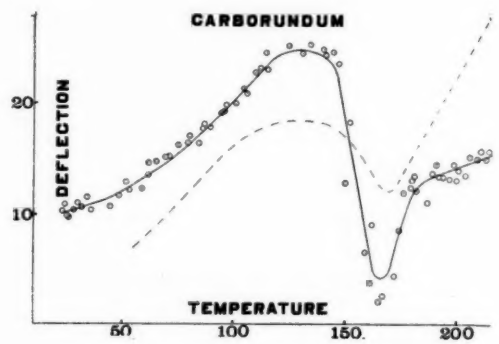


Fig. 10.

Upon heating the carborundum detector very peculiar results were obtained as may be seen in Fig. 10. Successive tests showed that the rectification attained a decided maximum at a temperature of about 130° C. and from there fell off rapidly until a minimum was reached at about 165° C. Above that temperature the rectification again increased as far as the tests were conducted. Even when the glass containing box accidentally broke and the detector was immersed in the hot oil-bath its behavior remained the same. This was probably due to the fact that the oil could not reach the actual contact between the crystal and the needle.

The cooling curve shown in Fig. 11 continued the lower portion of the heating curve. Rectification became zero at about $-85°$ C. It was noteworthy that upon being warmed the detector largely regained its rectifying properties, the dotted curve showing this rise. In order to prove that the decrease in rectification was not due to an imperfect contact caused by the contraction of the needle and the detector stand, we tried repeatedly to secure a new adjustment at these low temperatures but found it impossible. Likewise immersion in liquid air proved disastrous to all rectification.

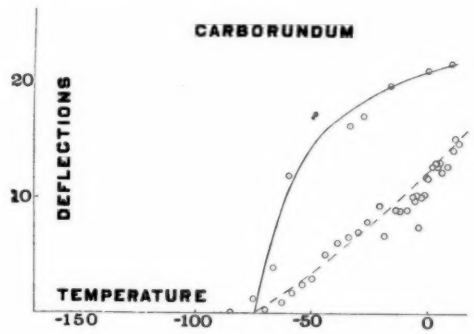


Fig. 11.

It was impossible to obtain any noticeable effects upon the rectification of carborundum by changes of humidity. It is probable that this was due to the imbedding of the contact point in the crystal so that the moisture could not affect it.

DISCUSSION.

It was found that with each of the four detectors, as far as we could observe, change in air pressure had no effect upon the rectification. We were able to vary the air pressure from 25 cm. to 135 cm. of mercury. It is perhaps not surprising that no effect was observed within this range.

One of the peculiarities noticed in our work was that certain crystals did not always rectify the oscillating current in the same direction. With one adjustment the rectified current would flow from crystal to point and with another would flow in the opposite direction. Once, indeed, during a test on the effect of change of temperature upon carborundum the rectification actually reversed although no change had occurred in the adjustment. The most common direction of rectification for silicon and for galena was found to be from the crystal to the brass point which was used. In the case of carborundum the ordinary sense of rectification was from needle to crystal. The perikon detector was the only one of the four which showed no reversal; always rectifying from chalcopyrite to zincite.

The "extra" E.M.F., to which reference has previously been made, is probably of galvanic origin. Although it might have been of thermo-electric origin, due to local heating at the contact, it could not have been the result of a thermo-junction caused by the difference in temperature between the detector and the outside circuit since both leads from the detector were of copper, causing the junctions affected by the heat to oppose each other. In addition the frequent reversal of this "extra" E.M.F. seems to preclude all possibility of its being due to a thermo-electric junction.

CONCLUSIONS.

It is possible that there are changes both in the condition of the surface on the crystal and in the body itself of the crystal during changes in the rectification. If both of these changes take place, as has been suggested by some observers, it is probable that the body effect predominates in the variations of rectification with changes in temperature which are most notable in the case of carborundum. In the case of humidity the surface effect probably has the most influence on the variation in rectification.

At certain times when the rectification was especially good, a humming sound was heard in the receivers similar to the inductive hum often heard when the detector circuit was opened. This seems to be evidence of the formation of a high resistance film on the surface of the crystal at the point of contact. Attention has previously been called to the possibility of the presence of this resistance film by Austin[1] and Goddard.[2]

There is apparently no relation between the rectification and the "extra" E.M.F. since the former remains unchanged through rapid fluctuations and often through reversals of the latter. The "extra" E.M.F. often exists even after room conditions of temperature, etc., have prevailed for some time throughout the apparatus. The cause of this "extra" E.M.F. and its relation, if any, to the rectification would be, in itself, a profitable subject for investigation.

We are indebted to Mr. Edison Pettit, of Washburn College, for suggesting this problem and to Professor F. E. Kester and the other members of the Department of Physics of the University of Kansas for their assistance and suggestions during the progress of the investigation.

BLAKE PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS,
May 12, 1915.

[1] Bulletin Bureau of Standards, Vol. 5, p. 146.
[2] PHYS. REV., 34, 423, 1912.

THE MAGNETIC SUSCEPTIBILITY OF GASES.[1] II.

BY W. P. ROOP.

8. The method of the gas manometer, as it may be called, has now been applied to the determination of the susceptibility of oxygen. Three features of the new determination distinguish it from previous work in the same field: observational errors have been reduced to values hitherto unattained; the observations have been made in fields of small intensity; and certain systematic errors of methods previously used have been eliminated.

Precision of measurements in this field has been very little discussed for the reason that previous work has shown wide discrepancies between successive observations taken by a given investigator, as well as between the averages obtained by different investigators. In Section 9 will be found a discussion of the work of Piccard,[2] the only previous observer who claims for his data a moderate degree of exactness.

The intensity of the field used is of importance for two reasons. In the first place, the measurement of a field of high intensity, but small extension, is a matter of considerable difficulty, and subject to undoubted errors for which allowance can not be made. In the second place, measurement of susceptibility in fields of both high and low intensity is necessary in order to obtain an experimental answer to the question: is suspectibility entirely independent of field intensity? This question will receive further consideration in Section 10.

Three systematic errors of other methods are absent in that of the gas manometer, namely: error due to the uncertainty as to the susceptibility of water; error due to imperfect determinations of pressure in the gas; and error due to solution of the gas in the liquid of the liquid manometer. These matters are discussed in detail in Section 13. In addition, the high sensitivity of the method reduces observational errors as well, since one source of the large errors of previous work has been the smallness of the quantities measured.

9. Piccard's paper unfortunately does not exhibit the data from which his mean values are calculated. It would appear, from data given on page 462 of his paper, that the probable error of an average of 15 deter-

[1] A continuation of work described in PHYSICAL REVIEW, 2, 497, 1913.

[2] Piccard, Archives des Sciences, 35, 476, 1913.

minations of the magnetic depression of water (Quincke's method) is 0.0028 mm. Since his determination of the susceptibility of oxygen is based on two such observations, it would seem to be subject to errors of that order of magnitude, or larger. On his page 479, the observed difference in magnetic depression of water due to presence of oxygen atmosphere is 0.033 cm. (misprinted in one place as mm.). The observational error in his mean result thus appears to lie between one and two per cent.

Observational error occurring in the use of the gas manometer is well under one per cent.

10. Systematic variation of susceptibility with field intensity has been observed by both Quincke[1] and Hennig.[2] A summary of data taken from their papers, and reproduced in the following table, shows this. The units in which susceptibility is expressed here and elsewhere in this paper, are ten millionths (10^{-7}).

TABLE I.

Data on Variation of κ with H.

Quincke.		Hennig.			
H	$\kappa \times 10^7$	H	$\kappa' \times 10^7$ 1 Atmos.	$\kappa \times 10^7$ 2 Atmos.	$\kappa \times 10^7$ 4 Atmos.
7,000	1.52	3,700	1.31		1.15
14,000	1.57	5,700	1.26		1.26
16,000	1.70	8,400	1.26	1.23	
		10,340	1.19	1.17	1.09

Both men, however, evidently considered their data insufficient to warrant even the suggestion that such a variation exists, for neither mentions the matter. The fact that the variation is in the opposite sense in the two cases is evidence of its spurious character. On account of the care used by Hennig, however, this should not have deterred him from calling attention to the indication given by his data. Instead of that, he excluded the two extreme values in casting up his definitive average.

Curie simply states, without giving data, that the susceptibility of oxygen was found not to vary within the limits of 100 and 1,350 gausses. These are much weaker fields, however, than those used by Quincke and Hennig, and his mean value is high, namely: 1.67. This was based on an assumed value for water of 7.9. The recent carefully determined value reached independently by Piccard and Sève is 7.2. This would convert Curie's value to 1.52.

[1] Quincke, Annalen der Physik, 34, 401, 1888.

[2] Hennig, Annalen der Physik, 50, 485, 1893.

Piccard's value of 1.407 was obtained with a more intense field (21,000 gausses) than that used by any of the others.

Langevin's theory of magnetization of gases demands a decrease of susceptibility with increasing field intensity, but according to his estimate, this should only begin to be perceptible at fields of the order of 100,000 gausses at atmospheric temperatures. The systematic variation shown by Hennig's data, and the discrepancy between the results of Curie and of Piccard would indicate that the field intensity at which saturation begins to be appreciable is not so high as that. A comparison of Piccard's work with the new results exhibited at the end of this paper may be considered to point to the same conclusion.

11. The apparatus used in the present experiments is the same in principle as that previously described, but quite different in design. It seemed impracticable to obtain a manometer of the desired form in glass. In order to reduce the time required for the enclosed gases to settle to their equilibrium position, it was necessary to make the bore of the manometer large and uniform. It also seemed desirable that it should be circular in shape, and that the two valves should be opened and closed simultaneously. The only feasible way of doing this was by using metal as a material. Since iron was excluded for magnetic reasons, mercury could not be used for handling the gas. It was therefore determined to make use of a vacuum pump and exhaust the manometer completely at each trial. The exhaustion demanded is about 0.01 mm., and difficulties were encountered in making the somewhat complicated brass manometer tight. This, however, was finally accomplished, and the present limitation to precision lies mainly in temperature control.

The manometer consists of a brass block containing a toroidal cavity of mean radius 46.7 mm., and cross-sectional area 33.33 mm.^{2} At opposite ends of a diameter, the toroid is divided by two ground stopcocks, connected by gear wheels so as to turn together. The diameter along which the toroid is thus divided is inclined at an angle of 20° with the horizontal.

Each chamber of the manometer may be connected separately with the vacuum pump, which is of the Toepler type. By means of this pump, the gas may be transferred to the burette, and its volume measured. On the way to the pump, the gas passes through a drying tube and an absorption tube filled with solid KOH.

The figure indicates schematically the arrangement of the apparatus.

12. The operations leading to a value for susceptibility may be summarized as follows: First, the cocks A and B, leading to the two chambers of the manometer, were opened, and the entire apparatus exhausted.

Cock C was turned so as to pass the gas into the atmosphere. At the same time the burette was completely filled with mercury by raising reservoir F until mercury flowed through cock C (now in its other position) into the chamber G. On reversing C again and raising reservoir E, all air was expelled from G. C was then closed and left closed. Cocks A

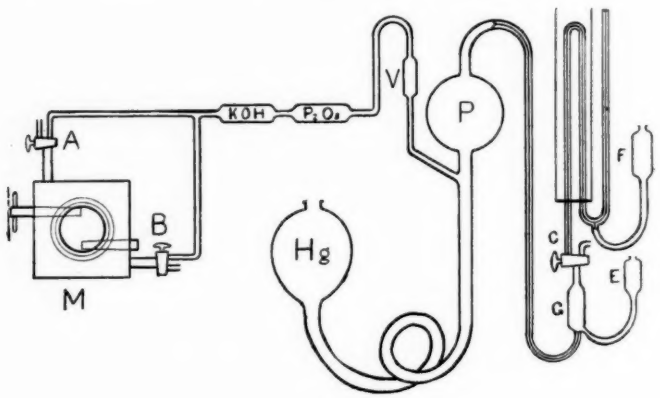


Fig. 1.

and B were now connected, respectively, with a supply of oxygen and of carbon dioxide. On opening the cocks A and B, the upper chamber of the manometer was thus filled with oxygen and the lower chamber with carbon dioxide. Now on closing A and B and opening the brass cocks of the manometer momentarily, the gases flowed around until the two boundaries between them lay at opposite ends of a horizontal diameter. Diffusion, of course, occurred at the same time. Difficulties thus introduced have already been treated in sections 4 and 5, and will be again discussed in section 17. After the manometer cocks were closed again, the upper chamber had received a small amount of carbon dioxide, and the lower chamber an equal amount of oxygen. Passing the gas into the burette, measurements of the volume of oxygen in each chamber were obtained.

The whole process was then repeated, with the exception that the two chambers of the manometer were connected with each other under the influence of the magnetic field. The boundaries between the two gases thus reached different equilibrium positions, and the volumes measured in the burette were different from those obtained before.

If the action of the field is to decrease the volume of oxygen left in the upper chamber, the volume of oxygen found in the lower chamber will be increased. This increase and decrease should be numerically equal, and in this way we obtain a valuable check on all observations.

We will call this observed deviation in volume ΔV. Divided by the cross-sectional area of the toroid, it gives the vertical displacement of the boundary surface due to the action of the field. This we will call l instead of d, as in Section 3.

The intensity of the magnetic field was determined by means described in Section 23.

The susceptibility difference between oxygen and carbon dioxide is given by the equation

$$\kappa - \kappa_0 = \frac{4lg(\rho - \rho_0)}{H^2 - H_0^2}.$$

13. Before proceeding to the discussion of the experimental details, I will call attention to certain broad features of the method.

In the first place, it is a comparison only, and not absolute in character. The same is true of all other methods that have ever been applied to the problem. In the cases of Quincke, Curie, and Piccard, the comparison substance was water, with respect to whose susceptibility an uncertainty of absolute value 0.5 exists, unless the work of Piccard and of Sève be accepted and all previous work rejected. In Quincke's method, used also by Piccard, error of the same absolute amount must be considered to attach to the value for oxygen, since it is the difference of susceptibility of oxygen and water that is measured. On the other hand, this uncertainty is reduced in case the oxygen is used under pressure greater than that of the atmosphere. Piccard does not state the pressure used, but only indicates that allowance for pressure was made in reducing data. Curie's method introduces uncertainty, not of the same absolute magnitude as that in the case of water, but in the same proportion. Hennig's method compares susceptibilities of oxygen and air, and is based on the assumption that the susceptibility of air is due simply to its oxygen content.

The adoption of carbon dioxide as comparison susbtance leads to uncertainty of the same order of magnitude as that which exists with respect to the susceptibility of that gas. I have given elsewhere[1] a summary of work on carbon dioxide which indicates that its susceptibility is certainly less than 0.03. Since different observers disagree, even as to sign, we may, in default of better knowledge, take its value to be zero. The uncertainty thus introduced does not exceed 2 per cent. of the susceptibility for oxygen.

I hope to completely settle this difficulty by the application of an absolute method now in hand.

[1] Physikalische Zeitschrift, 12, 55, 1911.

Another feature of the method of the gas manometer is that its indications are not affected by variations in pressure, except to the relatively small extent that specific susceptibility (susceptibility divided by density) depends on the state of compression. On the other hand, in Quincke's method, observed manometric depressions are in direct proportion to the pressure. This independence of pressure makes the method valuable for two reasons: no pressure correction is necessary; and the method is available for determining the variation of specific susceptibility with pressure, without diminution of precision at low pressures.

On the other hand, the method is rather sensitive to temperature changes, necessitating exact thermostatic control in an attempt to better the results here presented.

One type of difficulty inherent in Quincke's liquid manometer is completely avoided in the gas manometer, and that is the uncertainty as to the effect of solution of gas in the liquid. Even Piccard introduces no correction on this account, although an observation which he makes in another connection[1] would indicate that it is of considerable importance. From this difficulty Curie's work is exempt.

14. I will now proceed to discuss the following essential matters of detail in the present experiments: (*a*) Purity of gases; (*b*) Precautions in filling and operating manometer; (*c*) Diffusion in the manometer; (*d*) Temperature control; (*e*) Determination of sectional area of manometer; (*f*) Correction for curvature of manometer bore; (*g*) Field intensity measurements.

15. The gases were of the kind used for commercial purposes, stored in steel cylinders. The carbon dioxide was analyzed by absorption in the apparatus itself. In a number of trials, the largest volume of gas unabsorbed by solid KOH was observed to be 10 mm.^{3} out of a total of 5,300 mm.^{3} This quantity of air (the chief impurity) could neither cause an appreciable deviation from the density of pure carbon dioxide, nor affect the magnetic forces by its oxygen content. Therefore no correction was made for it.

The oxygen was obtained from the Linde Air Products Co., and was stated to contain about 2 per cent. of nitrogen. This figure was checked by a simple analysis. A glass bulb of 200 c.c. capacity was fitted with a stopcock at each end. It was filled with oxygen by running a steady slow stream of gas from the cylinder through it for five to ten minutes. While the gas was still running through, the lower end of the open tube was placed under the surface of a strong solution of KOH, and the upper cock was closed. The gas in the bulb was warmed with the hand, and a few

[1] Piccard, *loc. cit.*, p. 481.

bubbles driven out. When the temperature returned to that of the room, the KOH solution stood at an observed point in the bottom of the bulb. Pyrogallic acid was then poured in from above in such a way as to admit no air, at first only in small quantities. As absorption proceeded, the solution rose in the bulb, and more and more pyrogallic acid was added, until absorption was complete. The bulb was calibrated by water weighings. Two determinations yielded values of 98.03 and 97.84 for the percentage of oxygen in the gas. The mean was taken as the correct value. This somewhat crude, but very convenient analysis amply satisfies all demands of precision for the purpose in view.

16. The flow of gas from the cylinders was controlled by reducing valves, and the gas was admitted to the manometer through rubber tubes. Before each run, the rubber tube was detached from the cock and a slow steady stream directed against the opening in the end of the glass tube for two to ten minutes, depending on how recently the operation had been carried out before. The tube projecting from the cock was about 1 cm. long, and of capillary bore. It would appear that pure gas from the cylinder would thus reach the manometer, although failure to meet this condition may be partly responsible for the residual lack of uniformity in the observations.

Both gases were dried, and the oxygen was passed through KOH to insure the absence of carbon dioxide.

As soon as the rubber tube was placed over the end of the glass tube, the flow necessarily stopped, and the reducing valve maintained in the tube a steady, but small pressure. The cock (A, B) leading into the manometer was then opened, momentarily reducing the pressure, but not permanently. The reducing valve was then closed and a side cock opened for an instant to reduce the pressure in the tubes and chamber to that of the atmosphere. This, of course, was accompanied by a slight cooling of the gas in the manometer. After an allowance of time (three minutes) for the gas in the manometer to recover the temperature of the brass walls, the side cock was again opened momentarily, and the cock (A, B) leading to the manometer was then finally closed.

In spite of these elaborate precautions, the outstanding non-uniformity in observations seems to be largely due to failure to control the temperature, and hence pressure of the gas inside the manometer.

17. After the manometer had been filled, the brass cocks were opened for a certain length of time to allow the gases to settle into their equilibrium position. This time must be long enough not to interrupt the motion before the equilibrium position has been reached, and short enough to prevent diffusion beyond the limits of the uniform field while the cock still remained open.

To explain the means by which these conditions were satisfied, I will refer to the detailed drawing of the manometer. The broken circles represent the toroidal bore; *M* and *N* are the two cocks, with diagonal bore, as indicated.

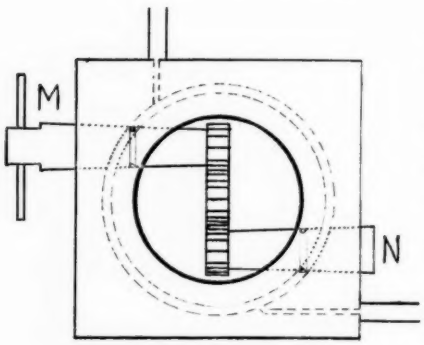


Fig. 2.

The bore of the manometer is large (diameter, 6 mm.) so as to reduce the viscous resistance to the motion of the gases. In order to reduce to a minimum the inevitable mixing during motion of the boundary, the distance through which the boundary must move is made as small as possible.

The time during which cocks *M* and *N* were left open was determined in accordance with indications as to diffusion obtained in the following way: a series of runs was carried through, with zero magnetic field, and with the cocks open for a length of time varying from 1 to 5 seconds. The results as exhibited in the table show that 1 second is certainly too little time, while 5 seconds is

TABLE II.

Table to Illustrate Diffusion.

Time, Secs.	1	1.6	2	2	3	5
Burette reading, mm.^{3}	464	532	535	529	532	559

certainly too much, since in that time an appreciable amount of oxygen has *diffused* into the lower chamber. The four observations running from 1.6 to 3 seconds, however, show no discrepancies exceeding the observational errors. As a result of this test, 2.5 seconds was taken as the standard length of time during which the cocks were held open. Exact determination of this time being unnecessary, opening and closing was done by hand, and time estimated to fifths of seconds by ear.

The question was also investigated, with similar results, with magnetic field on as well as off.

18. Certain details of design of the maonmeter were found essential to its proper operation.

Difficulty connected with the disposal of the gas contained within the rather large bore of the cock was settled as follows: a groove was cut around the plug in such a way as to intersect one of the open ends of the

bore of the plug. In this way, one of the plugs was connected with one chamber and the other with the other, in whatever position the plugs stood. The division of the gas in the manometer into two halves was thus sharp and without any ambiguity of any kind. The design of the plug is illustrated by the accompanying photograph.

In constructing the manometer, it was necessary to turn the cavity out in two plates which were then put together, face to face. The second photograph shows a section of the manometer. The large circle in the center is the section of the cavity, and the horizontal lines extending to each side are the sections of the faces of the plates. It was found

Fig. 3.

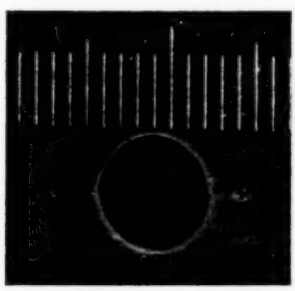


Fig. 4.

impossible in this way to make a permanently tight joint by simply soldering the two plates together, and recourse was had to an expedient which settled the difficulty satisfactorily. Just inside and outside the cavity, shallow grooves were cut in the faces of the two plates, and the sections of these may also be seen in the photograph. After the plates were soldered together, these grooves were squirted full of sealing wax. In order that the seats of the cocks might not be subject to the action of the exposed section of this sealing wax guard ring, they were somewhat enlarged and a bushing inserted. The cock plugs, which were of phosphor bronze, thus were ground into a uniform jointless brass surface, with satisfactory results.

19. The temperature of the brass manometer was closely observed by means of two thermometers, graduated to twentieths of a degree, whose bulbs were placed in cavities in the brass. The two thermometers rarely agreed exactly. The burette was water jacketed to retard temperature changes. Corrections for temperature difference between manometer and burette amounted, in the maximum, to 10 per cent.; in the average, to 2 per cent. Sufficient precautions were taken, however, to reduce uncorrected errors from this source well below 1 per cent.

Different runs were effected at different temperatures ranging from 13° to 20° C. No attempt was made to correct individual runs according to Curie's law, so that deviations from the average due to this cause are included with observational errors.

20. The bore of the manometer was determined by sawing a right section of it, and enlarging photographically. The area of the photograph (See Fig. 4) was determined by means of a planimeter. The scale shown in Fig. 4 is a scale of millimeters ruled on the brass surface with a dividing engine.

21. If the manometer were of uniform, vertical bore, l, the vertical deviation of the boundary due to the action of the field, would be simply ΔV, divided by the area of cross-section. Since the bore is circular, instead of straight and vertical, we must introduce a correction. Let a_0 denote the area of the normal section, and a that of the approximate horizontal section at the point at which the boundary rests in the deviated position. Referring to the figure, we see that

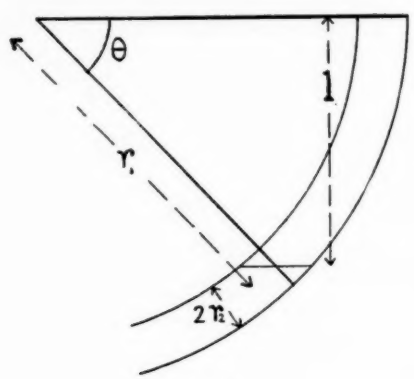


Fig. 5.

$$a = \frac{a_0}{\cos\theta}. \qquad (1)$$

Further,

$$\frac{l}{r_1} = \sin\theta,$$

whence

$$dl = r_1 \cos\theta d\theta.$$

Thus

$$adl = a_0 r_1 d\theta,$$

and

$$\Delta V = \int_0^l adl = a_0 r_1 \theta. \qquad (2)$$

To determine l, we therefore have the equation

$$l = r_1 \sin\theta = r_1 \sin\frac{\Delta V}{a_0 r_1}. \qquad (3)$$

This is based on the assumption that the boundary remains horizontal, and that equation (1) is exact. The latter condition is not satisfied, and we proceed to determine the error as follows: The true value of the horizontal sectional area we will denote by $a(1+\epsilon)$. ϵ will, of course, vary with l, but by obtaining a number of distributed values, and taking the average, we can determine a factor which can be taken out from under the integral and applied directly to the approximate result. To do this, we determine the exact area of horizontal cross-section as a function of l, r_1 and r_2.

Measuring x horizontally in the plane of the toroid, y vertically, and z normally to the plane of the toroid, we have, as the equation of the toroidal surface,

$$z^2 + (\sqrt{x^2 + y^2} - r_1)^2 = r_2^2.$$

The equation of the line of intersection with the plane

$$z = \pm l$$

is

$$z = \sqrt{r_2^2 - (\sqrt{x^2 + l^2} - r_1)^2},$$

and this is the curve bounding our horizontal section. To determine its area, we must evaluate the integral

$$\int_{r_1 - r_2}^{r_1 + r_2} z dx.$$

This I have not been able to do in general, but numerical evaluation at a series of values of l is simple and yields the desired result with sufficient exactness.

I find the maximum error to be 4.0 per cent., and the mean error, $\bar{\epsilon}$, 2.0 per cent., in the case where the correction is largest.

The corrected value of l is obtained from the equation

$$l = r_1 \sin \frac{\Delta V(1 - \bar{\epsilon})}{a_0 r_1}. \qquad (4)$$

22. In entire agreement with Piccard, I find that more careful attention is necessary in measuring the intensity of the field than has been given it in the past. A search coil and ballistic galvanometer were employed for the purpose. The coil was made by the builders of the magnet, the Société Genevoise, but the data accompanying the coil were found not to be trustworthy. The coil was calibrated by comparison with two precision coils, wound on glass. These two coils were constructed with great care and checked against each other to insure accuracy. Glass cylinders 20 × 6 mm. were ground true and calipered. Glass discs were cemented to their sides, and on each of the spools thus formed, a single layer, consisting of 25 turns of s.s.c. No. 34 copper was wound. The ends were specially arranged to avoid introduction of unevaluated magnetic area. The two coils were found by comparison in a field of 2,000 units to show a discrepancy of less than 0.2 per cent. The value of the area of the Geneva search coil as determined by comparison with the glass coils, may safely be stated to be in error by an amount not exceeding a similar figure.

Dependence was placed on the calculated area of the glass coils.

Experiment showed this to be far more satisfactory than any scheme of calibration of the coils in a calculated field.

The galvanometer was not in any sense used as a measuring instrument. It was calibrated carefully in connection with each reading with the search coils, at the identical deflections observed with the search coils. A standard solenoid and calibrated ammeter were used for the purpose.

23. I will now proceed to consider the data obtained. I give first in full the record of a sample pair of runs.

November 7.	Series 3, Run 37.		Page 217.
Manometer temp., 15.95.	I = 5.00 amps.		Cocks open 2.8 sec.
Upper Chamber.—Burette Readings,	4,664 mm.³		
Temp. of burette,	697		
16.55.	3,967		
Temperature correction,	8		
Unabsorbed gas,		3,959	
Lower Chamber.—Burette Readings,	5,143		
Temp. of burette,	3,050		
16.70.	2,093		
Temperature correction,	5		
Unabsorbed gas,		2,088	
Total unabsorbed gas,		6,047 mm.³	

November 7.	Series 3, Run 38.		Page 218.
Manometer temp., 14.90.	I = 0 amps.		Cocks open 3 sec.
Upper Chamber.—Burette Readings,	5,790		
Temp. of burette,	251		
15.35.	5,539		
Temperature correction,	9		
Unabsorbed gas,		5,530	
Lower Chamber.—Burette Readings,	3,628		
Temp. of burette,	3,120		
15.55.	508		
Temperature correction,	1		
Unabsorbed gas,		507	
Total unabsorbed gas,		6,037 mm.³	

ΔV:	5,530	2,088	
	3,959	507	
	1,571	1,581 mm.³	

24. In all, 140 runs like these were carried through. Of these, a great many, most of the first and second series, were rejected. Of the third series of 65 runs, a few also were rejected. No run was rejected by reason of its deviation in value for ΔV from other runs. The sole criterion was agreement as to total volume of unabsorbed gas.

There was a considerable unexplained variation in the value of this total volume of unabsorbed gas, but changes did not often occur suddenly. For this reason, readings at zero field were taken frequently, and com-

bination of successive runs in pairs like that shown above gives consistent results. When sudden variations did occur, they were taken as evidence of leakage, or failure to absorb perfectly, and the readings rejected. Attempts were made to correct on a basis of equalization of total volume of unabsorbed gas. The probable error was found to be somewhat reduced in this way, but the mean values did not change appreciably. Results as given are uncorrected.

25. Data from which the average ΔV for $I = 5.00$ amps. was obtained were as follows:

1,543
1,507
1,579
1,527
1,607
1,631
1,504
1,551
1,593
1,594
1,571
1,581
1,527
1,507
1,527
1,528
1,572
1,614

Mean value, 1,559 ± 10.

The averages at different field intensities are given in the table.

ΔV	I	H	No. of Obs.
1,559 ± 10	5.00	7,495	18
268 ± 5	2.00	3,405	17
75 ± 4	1.10	1,833	12
30 ± 7	0.65	1,080	12

26. Applying all corrections as indicated, I find

$$\text{at } H = 7{,}495,\ \kappa = 1.459 \times 10^{-7},$$

$$\text{at } H = 3{,}405,\ \kappa = 1.459 \times 10^{-7};$$

less trustworthy are the other two determinations:

$$\text{at } H = 1{,}833,\ \kappa = 1.41 \times 10^{-7},$$

$$\text{at } H = 1{,}080,\ \kappa = 1.6 \times 10^{-7}.$$

The agreement between the first two figures is partly coincidental. I

judge, however, that it is fair to state that the true value differs by less than 0.01×10^{-7} from 1.46×10^{-7}.

27. The discrepancy between this result and that of Piccard exceeds the estimates of probable error. This might be explained in four different ways. (*a*) The error may be larger than it is estimated to be. Experience shows that this is sometimes the case in work involving a number of possibilities of error. (*b*) The susceptibility of carbon dioxide may differ from zero. (*c*) Piccard's result may be in error by reason of failure to correct for dissolved oxygen, or by reason of error in the value for susceptibility of water. (*d*) Oxygen may show saturation phenomena at field intensities lower than Langevin's estimated limit.

The following table may be taken to give the best answer available at the present time to the question of magnetic saturation in oxygen.

Observer.	Field Intensity.	$\kappa \times 10^7$
Curie	100–1,350	1.52
Roop	3,400–7,500	1.46
Piccard	21,000	1.41

Further experiments in continuation of these here described are in progress. Acknowledgment is due the Whiting Fund, from which the magnet was purchased, Mr. W. R. Stamper, who constructed the manometer, and Mr. O. G. Steinitz, who constructed the standard search coils.

BERKELEY, CALIFORNIA,
December 24, 1915.

NOTE ON THE DAMPING FACTOR USED IN BALLISTIC CONSTANTS OF MOVING-COIL GALVANOMETERS.

By PAUL E. KLOPSTEG.

ATTENTION is called in a recent paper[1] to an error arising from the use of the factor $\sqrt{\theta_1/\theta_2}$ in obtaining from the throws of a damped galvanometer the equivalent throws under conditions of no damping.[2] An exact factor, $(\theta_1/\theta_2)^{2t_1/T}$, is given, where t_1 represents the time of the throw and T the complete period of the damped coil; and a curve is drawn from which its value may be obtained if θ_1/θ_2 is known. From the concluding paragraph of the paper cited one might infer that a direct determination of $2t_1/T$ is essential to the evaluation of the exact factor; on account of the difficulty of this determination the correction curve would therefore seem desirable.

In this connection it may, however, be well to note that the solution of the differential equation of motion of a galvanometer coil, under the usual assumptions, leads to the simple and theoretically exact expression for the ballistic constant

$$K = \frac{i}{\varphi} \cdot \frac{T_0}{2\pi} \cdot \rho^{\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}}, \qquad (1)$$

where i/φ is the current constant of the instrument,[3] T_0 the complete period of the *undamped* motion of the coil, ρ the ratio[4] of two consecutive swings and Λ the logarithmic decrement. For small values of Λ this equation becomes

$$K = \frac{i}{\varphi} \cdot \frac{\tau}{2\pi} \cdot \sqrt{\rho}, \qquad (2)$$

the degree of approximation being only 0.3 per cent. when Λ is as high as 0.184. Except for precise measurements this is sufficiently accurate. Equation (1) shows $\rho^{1/\pi \tan^{-1}(\pi/\Lambda)}$ to be the exact damping factor, which

[1] PHYS. REV., N.S., VI., 1915, 446: Lindley Pyle, "On the Correct Formula for the Damping Factor in Highly Damped Periodic Motion of the Coil of a D'Arsonval Galvanometer."

[2] Zeleny and Erikson Manual, 3d ed., 1912, p. 134; A. W. Smith, PHYS. REV., O.S., XXII., 1906, 250; Smith, Electrical Measurements, 1914, p. 147.

[3] In a paper not yet published the writer shows this factor, as ordinarily determined, to be the least accurate one involved in this equation. A method is also given for obtaining its value more exactly.

[4] ρ is identical with θ_1/θ_2 used above.

should be applied when Λ is greater than 0.184, or when higher accuracy is required in cases of smaller damping. Instead of using a curve to obtain from θ_1/θ_2 the value of this factor, it seems decidedly preferable to calculate it directly, since it depends only upon ρ, which is very easily determined. Or, to simplify matters still further, one might use the tables[1] which have been prepared for this purpose.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
January 10, 1916.

[1] Kohlrausch, Lehrbuch der praktischen Physik, 12th edition, 1914, p. 723; also Kohlrausch, 3d Eng. from 7th Ger. ed., 1894, p. 447.

THE DIFFRACTION RING PATTERN IN THE SHADOW OF A CIRCULAR OBJECT.

BY MASON E. HUFFORD.

DURING a series of experiments[1] in producing diffraction patterns by use of circular openings and steel spheres, it was observed that the concentric ring system produced by interference in the shadow of a disc or sphere was clearly brought out.

Lommel[2] has deduced formulas for calculating the radii of these rings and has compared the calculated and measured values for a few rings in microscopic diffraction patterns.

Since the number of rings here shown is much larger than that mentioned or shown by others,[3] an opportunity is offered to compute the radii by Lommel's formulas for many more rings and to compare computed and measured values over a much wider range.

In Lommel's expression,

$$M^2 = C^2 + S^2,$$

which represents the intensity of light at points in the geometric shadow, the integrals C and S involve quantities,

$$y = \frac{2\pi(a+b)r^2}{\lambda ab} \quad \text{and} \quad z = \frac{2\pi rR}{\lambda b},$$

in which a is the distance of a point source of monochromatic light of wave-length λ from the diffracting object of radius r, b is the distance of the object from the screen where the shadow is observed, and R is the distance from the center of the geometric shadow to the point in the shadow where the intensity of light is observed.

In order to determine the presence of points of maximum and minimum intensity of light in the shadow, the intensity M^2 is differentiated in the usual way with respect to a factor z. The result shows that maxima and minima of intensity will occur when,

$$V_0 = 0$$

where,

$$V_0 = J_0(z) - \left(\frac{z}{y}\right)^2 J_2(z) + \left(\frac{z}{y}\right)^4 J_4(z) - \cdots$$

[1] PHYS. REV., N. S., Vol. 3, No. 4, April, 1914.

[2] Lommel, Abhandlungen der Bayerische Academie der Wissenschaften, B. 15, 1886.

[3] Croft, Phil. Mag., S. 5, V. 38, 1894. Arkadiew, Phys. Zeit, 14, 1913.

and J_0 is the Bessel function. If y is large V_0 becomes equal to J_0 and the values of z for $J_0 = 0$ are approximately the values of z for $V_0 = 0$.

The values of y and z in $V_0 = 0$ plotted give a series of curves as shown in Fig. 1. The dotted lines are the graphs of $J_0 = 0$. The line $y = z$ marks the boundary of the geometrical shadow. All parts of the curves above this line apply to the shadow region. The parts of the curves which give minima are ruled heavy while the parts giving maxima are ruled light.

The possibility of using the values of z for which $J_0 = 0$ instead of those for which $V_0 = 0$ suggests itself.

The first ten of the values given in the following table of root values of the Bessel function were taken from tables. Beyond the tenth the values were calculated by McMahon's formula.[1] They are here given to the twenty-fifth value.

Z	Value of z by Formula.	Z	Value of z by Formula.	Z	Value of z by Formula.
Z_1	2.4048255577	Z_{10}	30.6346064684	Z_{19}	58.90712206
Z_2	5.5200781103	Z_{11}	33.7758991042	Z_{20}	62.04861462
Z_3	8.6537279129	Z_{12}	36.9171861194	Z_{21}	65.19011752
Z_4	11.7915344391	Z_{13}	40.05852068	Z_{22}	68.33162936
Z_5	14.9309177086	Z_{14}	43.19989382	Z_{23}	71.47314895
Z_6	18.0710639679	Z_{15}	46.34129753	Z_{24}	74.61450030
Z_7	21.2116366299	Z_{16}	49.49272626	Z_{25}	77.75602563
Z_8	24.3524715308	Z_{17}	52.63417501		
Z_9	27.4934791320	Z_{18}	55.76564161		

It is clear that if the value of y were infinite the values of z in $V_0 = 0$ would coincide with the values of z in $J_0 = 0$. Since the value of y is finite and rather small, the values of z corresponding to the value of y as they are taken from the graphs will differ more and more from the values of z in $J_0 = 0$ as they are taken from larger and larger graphs. It is clear that for the largest ring in any diffraction pattern, the value of z in $J_0 = 0$ will lack most of being the value of z for $V_0 = 0$. It is sufficient therefore to determine whether the value of z in $J_0 = 0$ may be substituted for the value of z in $V_0 = 0$ for the largest ring in any pattern.

This may be done if the error introduced is not greater than the experimental error.

To show how V_0 depends upon z we have:

$$\frac{dV_0}{dz} = -J_1(z)\left(\frac{z}{y}\right)^2 + J_3(z)\left(\frac{z}{y}\right)^4 - J_1(z)\left(\frac{z}{y}\right)^2 + \cdots,$$

[1] Annals of Mathematics, V. 9, 1894-5, p. 25.

since $J_0 = 0$ and,

$$J_n' = J_{n-1} - \frac{n}{z} J_n.$$

The Bessel functions with higher indices are decreasing quantities and also within the geometric shadow the value of y is always greater than that of z. Therefore the fractions will decrease rapidly and the expression is converging.

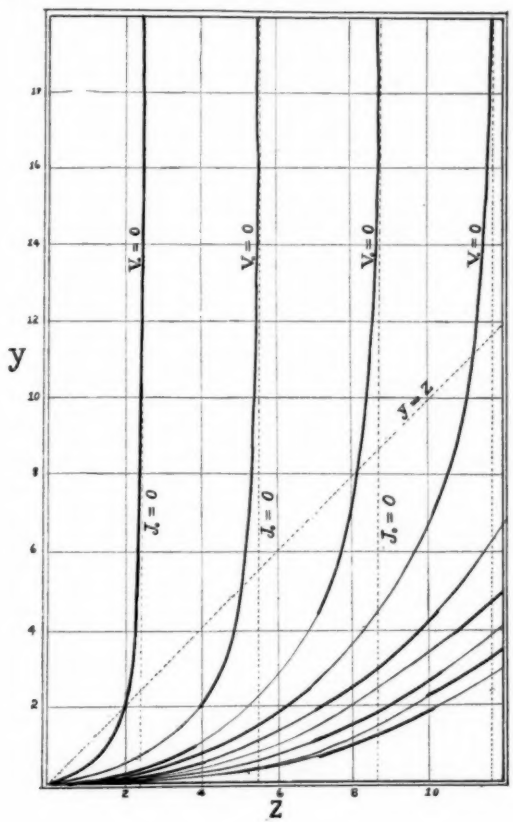


Fig. 1.

The accompanying photographs are the shadows of steel spheres, .317 cm., .6345 cm. and .9517 cm. in radius. These were obtained by apparatus similar to that described in a previous article.[1] The light from an arc was filtered through double thickness violet colored glass. From photographs of the spectrum of this filtered light it was observed that the most effective light which passed through the glass was that of wave-length, 3,880 A.U.

[1] See PHYS. REV., N. S., April, 1914, p. 243, Fig. 2.

By means of suitable lenses this light was focused on an opening .027 cm. in diameter. The light then traversed a box 3,255 cm. in length. At a distance of 1,550 cm. from the opening the spheres were suspended by means of very fine wires. The exposures were about six hours in length.

Substituting the values, $a = 1{,}550$, and $b = 1{,}705$ in the formula,

$$y = \frac{2\pi(a + b)r^2}{\lambda ab},$$

y for the smallest sphere is 20.02 and for the next larger one, $y = 80.23$.

The photographic negatives show that in the shadow of the small sphere, as shown in Photograph A, there are six rings representing minima of light intensity. In the shadow of the next larger sphere, as shown in Photograph D, there are about twenty-five rings representing minima of intensity. Photograph C is the shadow of the largest sphere used and Photographs B and E are enlargements.

The sixth and twenty-fifth roots of the Bessel function are, 18.0710639679 and 77.7560256302 respectively. These values apply to the sixth ring in the smallest pattern, Photograph A and to the twenty-fifth ring in the larger pattern, Photograph D. Whether these and smaller values may be used for calculating the radii of the rings depends upon how V_0 changes with respect to z in the immediate vicinity of these values.

If z is taken as 17 for the smallest sphere and as 75 for the next in size, then dV_0/dz for the former is .268 and for the latter it is, .2718. If the actual values of z could be found they would probably be larger than these and therefore the rates of variation would be less.

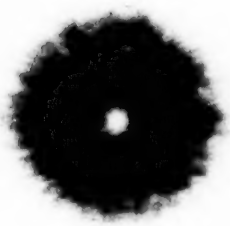
For calculating the first of these rates, the values of J_1, J_3, J_5, etc., were taken from tables in Gray and Matthew's Treatise on Bessel Functions. For calculating the second of the rates, the corresponding values of J_1, J_3, etc., were found by formulas given in Gray and Matthews, pp. 40 and 13.

These values are given below:

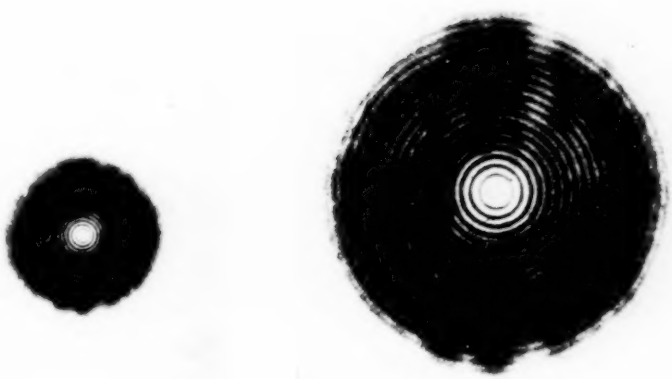
$J_1(75)$	-0.085137365	$J_5(75)$	-0.078166306	$J_9(75)$	-0.0540197166
$J_2(75)$	-0.03691648	$J_6(75)$	-0.0573188674	$J_{10}(75)$	-0.0834193863
$J_3(75)$	$+0.083168445$	$J_7(75)$	$+0.0689952873$	$J_{11}(75)$	$+0.0317745466$
$J_4(75)$	$+0.046896693$	$J_8(75)$	$+0.0701979877$		

Six terms of the expansion, dV_0/dz were used in calculating the rates of variation.

A. B.



C.



D. E.

MASON E. HUFFORD.

The results of several trials showed that there was an error of about one per cent. in setting the cross hair of the measuring instrument on the transparent rings of the negatives.

Since the rather large rates of variation of V_0 with z apply only to the last rings of the patterns, it was thought sufficiently accurate to use the root values of z for calculating the radii of the diffraction rings.

The calculated and measured values are given below with the difference between them expressed as a per cent. of the calculated value.

$a = 1{,}550$ cm. $\quad b = 1{,}705$ cm.

$\lambda = 3{,}880$ A.U. $\quad D = .027$ cm.

Sphere No. 1. *Radius* .317 *cm.* $y = 20.02$.

No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.
1	.079	.080	+1.2
2	.183	.184	+0.5
3	.287	.287	+0.1
4	.391	.386	−1.4
5	.485	.483	−0.6
6	.600	.575	−4.0

Sphere No. 2. *Radius* .6345 *cm.* $y = 80.23$.

No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.	No.	Calculated Cm.	Measured Cm.	Diff. in Per Cent.
1	.039	.042	+7.0	13	.663	.652	−1.6
2	.091	.092	+1.5	14	.715	.704	−1.5
3	.143	.144	+0.5	15	.767	.755	−1.6
4	.195	.194	−0.3	16	.819	.802	−2.1
5	.247	.243	−1.7	17	.871	.858	−1.4
6	.299	.292	−2.4	18	.923	.910	−1.8
7	.351	.344	−2.0	19	.975	.958	−1.9
8	.403	.396	−1.8	20	1.027	1.013	−1.3
9	.455	.436	−4.2	21	1.079	1.062	−1.6
10	.507	.499	−1.8	22	1.132	1.113	−1.7
11	.559	.549	−1.7	23	1.184	1.162	−1.8
12	.611	.601	−1.6	24	1.236	1.208	−1.4

The above calculated values of the rings were found from the formula:

$$R = \frac{2\pi r}{\lambda b z}.$$

The measured values are the averages of four independent measurements of each radius. These were made by means of a comparator.

While it was possible to count twenty-five rings on the larger negative without the aid of a microscope it was found that there was so much lack

of detail near the edge of the shadow that the cross hair could not be set on the outermost ring with any degree of accuracy. Twenty-four of the twenty-five rings were therefore measured.

In conclusion I wish to express my thanks to Prof. A. L. Foley, who has made many valuable suggestions throughout the progress of this work. I wish also to thank Prof. R. D. Carmichael, formerly of Indiana University, for valuable hints in the mathematical part of the work. I am indebted to Indiana University for placing the apparatus at my disposal and to the Chicago University Library for the loan of useful books.

LABORATORY OF PHYSICS,
INDIANA UNIVERSITY,
December, 1915.

THE EFFECT OF TEMPERATURE ON THE RESISTANCE, THE LIGHT-SENSITIVENESS, AND THE RATE OF RECOVERY OF CERTAIN CRYSTALS OF METALLIC SELENIUM.

BY KATHRYN JOHNSTONE DIETERICH.

THE purpose of this article is to set forth certain facts concerning the effect of variation of temperature on the rate of recovery after illumination, the sensitiveness to light, and the resistance of certain crystals of metallic selenium.

APPARATUS.

The apparatus used is shown in Fig. 1. The crystal was placed between platinum electrodes, *E*, such that the weight, *W*, suspended from the upper electrode, maintained a constant pressure on the crystal.

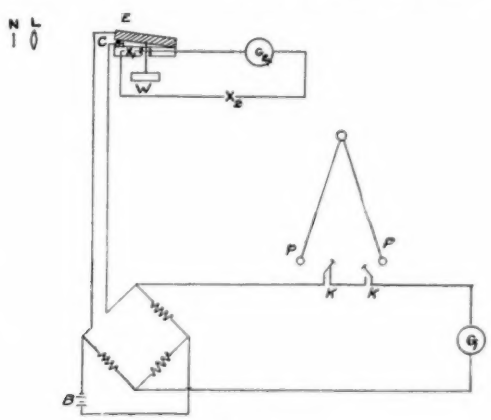


Fig. 1.

The source of illumination was a Nernst glower, *N*, at a distance of 45 cm., the intensity of illumination being kept constant by maintaining over the lamp a voltage of 115 volts from storage batteries. The crystal constituted one arm of a Wheatstone bridge circuit, over which a constant voltage of 10 volts was kept. As nearly as possible all conditions of light intensity, pressure, voltage, and time of recovery were kept constant and only temperature varied. The latter was measured by means of a copper-constantin thermocouple in series with a galvanometer. One

junction, X_1, was placed beside the crystal, the other, X_2, was kept at 0° C.

In carrying out this investigation it was necessary, at each temperature, to measure the resistance of the selenium when it had reached equilibrium in the dark, in the light, and after it had been in the dark the desired short interval of time, viz., .02 sec. The first two measurements were made by the usual balanced bridge method, the last by the method of Brown and Clark.[1] This consisted in balancing the Wheatstone bridge circuit with the selenium in equilibrium in the light, and then opening the galvanometer circuit. A timing pendulum, which had been arranged to operate a metal shutter to cut off the light from the crystal and throw the galvanometer in circuit simultaneously, was then released. A deflection of the galvanometer resulted, due to the change in resistance of the crystal when the light was removed. The resistance of the crystal at the end of .02 sec. in the dark was determined, later, by substituting for the crystal a carbon resistance, which could be adjusted, until upon releasing the pendulum, the same galvanometer deflection was obtained as had been given by the crystal. The carbon resistance, since it was constant, was then measured by balancing the bridge circuit.

THEORETICAL.

When a crystal of metallic selenium has been exposed to light, and the light is suddenly removed, the conductivity of the crystal does not return instantly to the value of the equilibrium dark conductivity, but decreases rapidly at first, then more slowly, requiring several minutes or even hours to attain its final value, depending on the temperature. When a gas is exposed to a source of ionization, and the source is suddenly removed, the conductivity decreases rapidly at first, then more slowly, until equilibrium is restored. In gases the decrease in conductivity during the first short interval of time is ascribed to the recombination of the conducting electrons with the positive residues. It has been shown by Rutherford,[2] McClung,[3] McClelland[4] and more recently, Plimpton,[5] that if there are present n_1 negative ions and n_2 positive ions, the rate of recombination is directly proportional to $n_1 \times n_2$, that is

$$\frac{dN}{dt} = -\alpha n_1 \times n_2,$$

where α is a constant and is called the coefficient of recombination.

[1] PHYS. REV., Series 1, XXXIII., p. 53, 1911.
[2] Phil. Mag., Series 5, Vol. 44, p. 422, 1897; Vol. 47, p. 109, 1899.
[3] Phil. Mag., Series 6, Vol. 3, p. 283, 1902.
[4] Phil. Mag., Series 5, Vol. 46, p. 29, 1898.
[5] Phil. Mag., Series 6, Vol. 25, p. 65, 1913.

In order to study the effect of pressure, light intensity, and electromotive force on the rate of recovery in selenium, in cells and crystals, Professor Brown[1] assumes that for the first short interval of time after the light has been removed the decrease in conductivity of a selenium cell or crystal is due to the recombination of the conducting electrons with positive residues, and that this recombination takes place according to the same law as that which holds for gases. If the number of electrons taking part in the conduction is proportional to the conductivity then the rate of recombination may be calculated from the formula:

$$\alpha_1 = -\frac{\Delta C}{C^2 \Delta t},$$

where ΔC is the decrease in the conductivity during the time, Δt, that the crystal has been in the dark, C is the equilibrium value of the conductivity in the light, and α_1 is proportional to the coefficient of recombination. In order to obtain this result, Professor Brown found it necessary to make the following assumptions:

1. When the crystal is in equilibrium in the light or in the dark the conductivity is proportional to the number of negative electrons taking part in the conduction.

2. Recombination takes place according to the same law as in gases.

3. The rate of recombination is the same in the light as in the dark.

The above conception was verified approximately by varying the conductivity over a wide range. This was done by changing the intensity of illumination. Since the rate of recombination was found to be nearly proportional to the square of the conductivity it was assumed that the recombination followed the same law as in gases.

In this paper the results of an investigation of the effect of temperature on the rate of recovery of selenium crystals will include a discussion of the effect of temperature on the coefficient of recombination, calculated on the above theory.

RATE OF RECOVERY.

The crystals used were prepared by Dr. Brown[2] as described by him. They were of three kinds: the large hexagonal, the needle shaped acicular, and the monoclinic lamellar. In each case the current was passed through the crystal in a direction perpendicular to the direction of the long axis, and the crystal illuminated in a direction perpendicular to the direction of the current.

In general the coefficient of recombination was found to decrease with increase in temperature.

[1] PHYS. REV., Series 2, Vol. 5, p. 395, 1915.

[2] PHYS. REV., Series 2, Vol. 5, p. 236, 1915; Vol. 4, p. 85, 1914.

TABLE I.

Temperature in °C.	Temperature in ° Abs.	Conductivity in Light. $Ohms^{-1} \times 10^7$.	$\alpha \times 10^{-7}$.	$\alpha \times C$.
−89	184	1.685	3.640	6.11
−68	205	2.057	3.280	6.75
−40	233	3.580	1.680	6.02
0	273	7.070	0.875	6.17
19	292	7.460	0.835	6.22
21	294	7.880	0.754	5.93
26	299	8.070	0.730	5.89
33	306	8.660	0.606	5.23
43.5	316.5	9.070	0.548	4.95
56	329	9.310	0.537	4.99
64	337	5.890	0.367	2.17
75	348	8.320	0.316	2.62

Table I. shows the results obtained with hexagonal crystal No. 2. There is seen to be a large fall in the value of α with increase of temperature from − 89° C. to about 0° C., after which the decrease is more gradual to 75° C. which is as high a temperature as could be reached. Upon examining the electrodes it was discovered, early in the work, that at the higher temperatures a dark deposit was formed between the

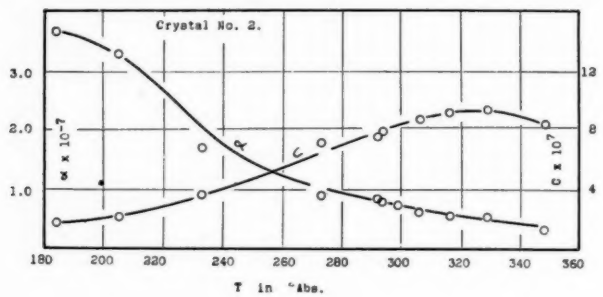


Fig. 2.

crystal and the electrodes. The effect of this deposit was to depress both the values of conductivity and of α, so that the last two points on the curve, Fig. 2, showing the relation between α and temperature, are only approximately correct.

Crystal No. 13, Table II., shows in general the same result; a large decrease in α with increased temperature. In this case the temperature was not high enough to produce sufficient deposit to cause any appreciable error.

The curve, Fig. 3, shows a slight depression in the values of α at 10° C., which may or may not be a genuine effect. However, two complete

series of readings gave a low value of α at just this temperature. It may be of interest to mention here that, although a crystal had been removed from the electrodes for several weeks, it could be returned and, if the

TABLE II.

Crystal No. 13. E.M.F. = 6 volts. Pressure = 75 gr.

Temperature in °C.	Temperature in ° Abs.	Conductivity in Light. $Ohms^{-1} \times 10^7$.	$\alpha \times 10^{-7}$.	$\alpha \times C$.
−40	233	2.63	2.010	5.26
− 7	266	4.35	0.925	4.03
10	283	7.10	0.460	3.26
21	294	6.05	0.548	3.31
32	305	6.82	0.398	2.73
41	314	7.38	0.348	2.57

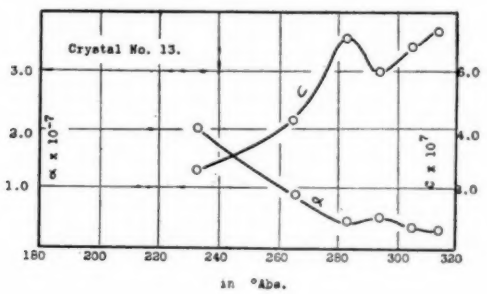


Fig. 3.

previous conditions as to light intensity, pressure, and voltage were restored, the same curve of α and temperature was obtained, giving the same values of α as before.

Other crystals showed essentially the same change of α with temperature. In some cases there were depressions as in No. 13; one crystal, No. 3, a lamellar type, showed quite a sharp kink at 35° C. Thus, each of the three types of crystals showed the same effect of temperature on the coefficient of recombination: a general decrease in α with increase of temperature.

H. A. Erikson,[1] in 1909, made a determination of the effect of temperature upon the coefficient of recombination in gases. He obtained a curve showing the relation between α and T which resembles very closely the curve obtained for crystal No. 2, Fig. 2. His curve is shown in Fig. 4. However, no definite significance can be attached to this striking similarity as a result of this investigation only.

[1] Phil. Mag., Series 6, Vol. 18, p. 328, 1909.

If then, the recovery of selenium crystals is due to a recombination of negative electrons with positive residues, and the rate of recombination is expressed by the formula,

$$\frac{\Delta C}{\Delta t} = -\alpha C^2,$$

the coefficient of recombination, α, varies with temperature.

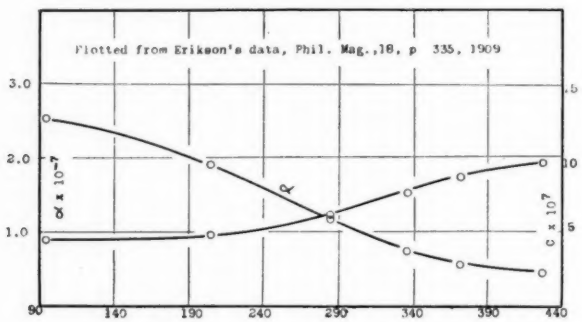


Fig. 4.

It will be noticed, in Tables I. and II., that, while the coefficient of recombination is not a constant at all temperatures, the product, $\alpha \times C$, is very nearly so. A glance at the curves, Figs. 2 and 3, will show that this relation suggests itself here also; since wherever a rise occurs in the α-curve a depression appears in the C-curve. The same general relation between the shape of the α-curve and the C-curve is apparent in Erikson's work in gases, Fig. 4. Dr. Brown[1] has found that $\alpha \times C$ is a constant for selenium crystals when pressure or voltage varied the conductivity, although α is not constant.

Since $\alpha \times C$ is a constant and is equal to $\Delta C/C \times \Delta t$, the rate of recovery during the first short interval of time is equal to some constant, β, times the conductivity in the light.

$$\Delta C/\Delta t = \alpha C^2 = \beta C;$$

where β is equal to $\alpha \times C$, and we have the simple relation that the rate of recovery of selenium crystals is proportional to the conductivity in the light. This proportionality did not appear to hold with much accuracy at the higher temperatures.

[1] *Loc. cit.*

RESISTANCE.

Conductivity is determined by taking the reciprocal of the resistance, so that a study of the effect of increase of temperature upon the conductivity of selenium crystals should give an adequate idea of its effect on their resistance. In crystal No. 2, Fig. 2, it will be seen that the conductivity in the light at low temperatures was small, increased with increasing temperature, and passed through a maximum in the region of 55° C. The fall of conductivity above 55° C. might have been influenced by oxidation of the brass electrodes which were used with this crystal, so that the apparent decrease was somewhat exaggerated. Crystal No. 13, Fig. 3, showed a general increase of conductivity in the light with increased temperature. There was, however, a decrease for a short interval from 10° C. to 21° C., after which the conductivity rose again as the temperature increased. Depressions of this kind were found in the *C*-curves of many of the crystals, although not always at this same temperature. One crystal, No. 4, had a sharp depression and sudden rise in the region of 0° C. Ries[1] found the same condition to hold for selenium cells, with the minimum most often around 100° C., but that he was able to shift the position of this minimum to below 0° C. by certain heat treatment. The general tendency, however, in cells and crystals is for an increase of conductivity, and a consequent decrease of resistance, to result from an increase in temperature.

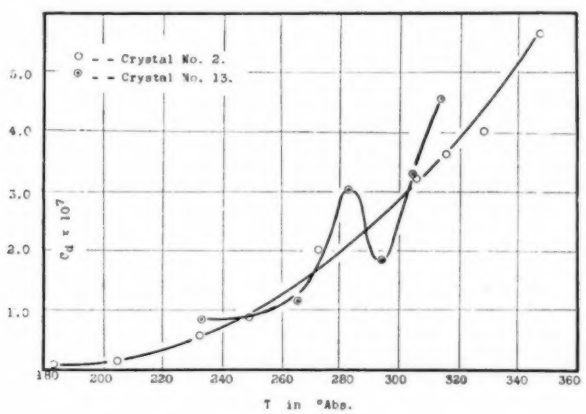


Fig. 5.

The equilibrium dark conductivity will be seen from the curves in Fig. 5 to follow much the same general tendency: an increase of conductivity with increase of temperature. The same maxima and minima which were found in the light conductivity curves are apparent here.

[1] Die Elektrischen Eigenschaften und die Bedeutung des Selens für die Electrotechnik, 1908.

LIGHT SENSITIVENESS.

The definition of light sensitiveness may be formulated arbitrarily to best meet the needs of practical use, as in the operation of galvanometers and relays, or it may arise from purely theoretical considerations arrived at in an attempt to explain the effect of light on crystals. Several of the more common expressions for light sensitiveness determined by practical usage are,

1. The ratio of the change of resistance to the resistance in the dark. $(R_d - R_l)/R_d$.

2. The ratio of the resistance in the light to the resistance in the dark. R_l/R_d.

3. The difference between the conductivity in equilibrium in the light and in the dark. $C_l - C_d$.

In the third case it is necessary to know not only this difference but also the terms which produce it, namely, C_l and C_d, to form an adequate idea of the possibility of adapting the crystal to the work at hand. For the same reason one would need to know the intensity of the light producing this change.

However, the theory which underlies this paper makes it reasonable to define sensibility in still another way, namely, in terms of the rate of production of conducting electrons. Dr. Brown[1] has shown how this rate of production may be calculated.

$$\frac{dC}{dt} = M + q;$$

where dC/dt is the rate of production of electrons when the crystal is in equilibrium in the light; q is the rate of production in the dark and M is the rate of production by the light. dC/dt must also be equal to the rate of recombination, αC^2, since the crystal is in equilibrium. For the first few hundredths of a second after the illumination is removed

$$dC/dt = \alpha C^2 = M + q.$$

When the crystal is in equilibrium in the dark the rate of production of electrons, q, is equal to the rate of recombination, αC_d^2, and M is zero. When the crystal is in equilibrium in the light

$$\Delta C/\Delta t = M + \alpha C_d^2;$$

$$M = \Delta C/\Delta t - \alpha C_d^2$$

$$= \alpha C_l^2 - \alpha C_d^2.$$

[1] *Loc. cit.*

TABLE III.

Crystal No. 2, in ° Abs.	C_l (Ohms^{-1} × 10^7.)	C_d(Ohms^{-1} × 10^7.)	$C_l - C_d$ (Ohms^{-1} × 10^7.)	M
184	1.685	.106	1.58	10.2
205	2.057	.135	1.92	13.8
233	4.190	.583	3.61	26.8
273	7.070	2.150	4.82	32.47
293	7.880	2.280	5.60	34.92
299	8.070	3.030	5.04	35.37
306	8.660	3.230	5.43	39.10
316	9.070	3.620	5.45	38.00
326	9.310	4.080	5.23	37.80
348	8.320	5.670	2.65	11.60
Crystal No. 13.				
233	2.630	.588	2.042	12.21
266	4.350	1.150	3.20	16.25
283	7.100	3.030	4.07	19.08
294	6.050	1.830	4.22	18.17
305	6.820	3.280	3.54	14.29
314	7.380	4.550	2.83	11.80

Table III. and Fig. 6 show the effect of temperature upon light sensitiveness as defined by the practical working definition, $C_l - C_d$. It will be seen that the sensitiveness at first increases with increase of tempera-

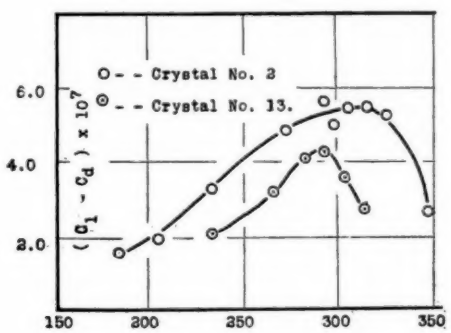


Fig. 6.

ture to about the region of room temperature after which it decreases. Prof. L. P. Sieg[1] has noted that selenium plates show a decrease in sensitiveness to light, from room temperature to 100° C. when the light sensitiveness is defined as the ratio of conductivity in the light to the

[1] PHYS. REV., Series 2, Vol. 6, p. 213, 1915.

conductivity in the dark. Fig. 7 shows the same general relation between temperature and light sensitiveness, when the latter is measured by the rate of production, M, of electrons by the light.

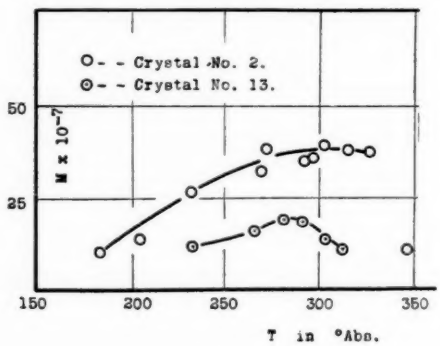


Fig. 7.

SUMMARY.

In conclusion, it may be said in regard to the effect of temperature on certain properties of selenium crystals, that,

1. The resistance in the light and in the dark is, in general, decreased by increase of temperature, with a small increase in some cases in the region 0° C. to 50° C.

2. The coefficient of recombination, α, calculated according to the formula,

$$\alpha = -\Delta C/C_1^2\Delta t,$$

is not constant at all temperatures but decreases with increase of temperature in much the same manner that the coefficient of recombination decreases in gases with increase in temperature.

3. Over a wide range of temperatures the rate of recovery for the first few hundredths of a second is proportional to the conductivity in the light.

$$\Delta C/\Delta t = -\beta C_1.$$

4. The sensitiveness to light does not vary greatly over a wide range of temperature, although a tendency toward a maximum is found in the region 30° C. to 50° C.

I wish to acknowledge my indebtedness to the staff of the Department of Physics for their interest in the problem, and especially to Dr. Brown for suggesting it, and for his encouragement during the progress of the work.

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THE STATE UNIVERSITY OF IOWA.

SUB-HELMHOLTZIAN VIBRATIONS OF A RUBBED STRING.

BY HARRY CLARK.

I. STATEMENT OF THE PROBLEM AND REVIEW OF PREVIOUS INVESTIGATIONS.

THE problem of the longitudinal vibration of a rubbed string has been extensively studied by Davis,[1] to whom it was suggested by its usefulness in the study of the effects of mechanical stress upon the magnetic properties of materials. The writer has undertaken a further study of the subject with a two-fold purpose: first, of determining more carefully the conditions necessary for the production and accurate duplication of the various wave forms with unfailing regularity; and second, of studying in detail particular conditions of rubbing hitherto uninvestigated.

A satisfactory solution of the first of these problems is due largely to the adoption of a new rubbing device whose action is extremely uniform and reliable. The second problem refers to the so-called sub-Helmholtzian vibrations, which under favorable circumstances may be produced by rubbing with slight pressure. These waves were investigated by means of an improved photographic method of study. Several features of interest are presented by them.

In addition to the study of sub-Helmholtzian waves, the experiments to be described have verified nicely some of the previous work on Helmholtzian forms, and have brought out a new criticism of the laws governing them. A complete understanding of the work will be facilitated by reviewing briefly at this point some of the studies of transverse vibration.

The problem of the bowed string cannot be successfully attacked by the use of analysis without a complete knowledge of the nature of the bowing process. Much of this knowledge is embodied in the following laws:

Young's Law.[2]—No overtone is present which would have a node at the point of excitation.

Helmholtz's Law.[3]—When a string is bowed at an aliquot point $(1/q)$, the part of the string immediately under the bow moves to and fro with

[1] Proc. of the Am. Acad. of Arts and Sci., Vol. XLI., No. 32, May, 1906.

[2] Phil. Trans. of the Roy. Soc. of London; 901, 106, 1800.

[3] Proc. Glasgow Phil. Soc., Dec. 19, 1860; reprinted in Phil. Mag., Ser. 4, 21, 393, 1861.

constant velocities whose ratio is equal to the ratio $(1/(q-1))$ of the segments into which the string is divided by the point in question.

Helmholtz's Velocity Law.—The smaller of these two velocities has the same direction as that of the bow and is equal to it.

From these facts Helmholtz concluded that the action of the bow is accounted for by something analogous to the difference between the coefficients of static and moving friction. That wave will be developed which allows the element of string under the bow to travel as much of the time as possible, in the direction of the bow's motion, without slipping.

On the basis of these laws Helmholtz solved completely all aliquot cases of the problem. His solution is given by

$$u = u_1 - u_2,$$

where u_1 and u_2 are defined by

$$u_1 = A \sum_n \frac{1}{n^2} \sin \frac{n\pi y}{l} \sin \frac{n\pi t}{T}, \tag{1}$$

$$u_2 = A \sum_m \frac{1}{q^2 n^2} \sin \frac{qm\pi y}{l} \sin \frac{qm\pi t}{T}, \tag{2}$$

u represents the displacement of a point, y, on the string at time, t. The length of the string is l, and the period of a complete vibration is $2T$, m and n are integers, A is constant for a given speed of bowing, and q has been defined above. Equation (1) is called Helmholtz's *major* solution. The value of u, determined by it, is seen to be independent of q, which defines the place of bowing. Equation (2), the *minor* solution, serves, for any particular value of q, to exclude those partials whose presence would violate the law of Young. It becomes relatively unimportant for large values of q.

Rational bowing points, which include the aliquot cases of course, may be solved if the law of Helmholtz be replaced by

Krigar-Menzel's Law.[1]—When a string is bowed at any rational point (p/q), where p and q are prime to each other, the part of the string immediately under the bow moves to and fro with constant velocities whose ratio depends only on q and is $1/(q-1)$.

These four laws constitute the established theory of rational cases of strings transversely bowed. No investigations of other cases have been published. It is to be noted that the solutions which have been obtained are considered to be independent of the nature of the bow and of the force with which it is pressed against the string. All waves obeying the above laws, whether transverse or longitudinal, are called *Helmholtzian waves.*

[1] Inaugural Dissertation, Berlin, 1888.

Although the theories of transverse and longitudinal waves are very much alike, it is desirable to note some of the differences between the two classes of phenomena in order to see for what reasons a particular interest attaches to the study of longitudinal forms.

Sharp corners of transverse waves are quickly rounded off by the quality of "stiffness" in the string. In the case of longitudinal vibrations, however, these sharp corners are replaced by discontinuities of density. Such discontinuities are not difficult of conception, and there is no evidence to show that they exercise any distorting influence on the waves. On the theoretical side therefore the longitudinal is the simpler type. Since the frequency of longitudinal vibrations involves only the nature of the material in the string and the length of the vibrating portion, it remains very nearly constant. The operation of longitudinal "bowing" may by mechanical means be made continuous, so that a given state of vibration may be maintained indefinitely.

The very complete investigation by Davis of longitudinal Helmholtzian waves is briefly reviewed below.

He used for the "string" a steel wire 4 mm. in diameter and somewhat less than 7 meters in length, with ends firmly clamped to massive iron blocks. A knife edge, pressed lightly against the wire at the middle, divided it into two equal segments, one of which was reserved for observation. In the other was placed the "bow," consisting of a motor-driven wheel, of which the face was covered with well-resined chamois skin.

The methods of observation were two. In the first, a very small glass ball, perhaps 1 mm. in diameter, was cemented to the wire, brilliantly illuminated from a point source, and observed through a microscope. From the limits of its motion the amplitude of vibration was obtained. Such amplitudes, measured at many points along the wire, and plotted as though the motions were transverse, gave the envelope of the wave form. In the second method, a short piece of a needle was attached to the wire, and the point was allowed to trace a record of its motion upon a smoked glass drawn transversely underneath. Although the amplitudes employed were very small, only a few tenths of a millimeter, very perfect tracings were obtained.

By the experiments it was shown that all of the laws of transverse vibration apply without change to the longitudinal forms. A large number of cases were investigated, and the verification is quite complete.

Among other contributions of Davis is the *integral surface*, a graphical construction designed to facilitate the study of both longitudinal and transverse Helmholtzian waves. It gives the motion of all points on the string if the motion of one point be completely known. Since any

curve may be represented approximately by a broken line of straight segments, the usefulness of the integral surface may be extended to types of waves other than Helmholtzian. The original paper[1] should be consulted for a complete description of the method.

Davis discovered that when, under certain circumstances not well understood, the pressure of the rubbing wheel against the wire is decreased, a point is reached below which the waves, though quite stable, cease to be Helmholtzian. Their amplitudes (observed by his first method only) were found to be too small to fit Helmholtzian theory. In some instances there were maintained waves with amplitudes scarcely more than six-tenths as great as those of high-pressure waves. To distinguish waves of this type they are called *sub-Helmholtzian*.

II. Apparatus and Methods of Use.

The string used in these experiments was a steel wire .101 cm. in diameter, of the quality commonly known as "music wire." It was carefully straightened and nickel-plated. To secure consistent results it was found that the use of a simple vibrating segment is preferable, although the difficulties of observation are thereby increased.

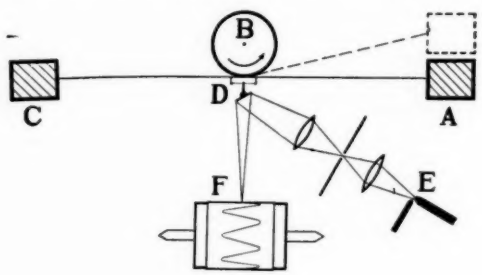

Fig. 1.

Thus arranged the vibrating portion of the wire is ten meters long. This gives a frequency of 254 vibrations per second corresponding to a pitch very nearly that of C_3. The low frequency of vibration allows the recording mechanism to work under more favorable conditions.

The wire was stretched horizontally, as shown in Fig. 1. One end is held firmly in the jaws of a vise, A, weighing about 6 kg., which can be moved by a feed-screw horizontally in a direction perpendicular to that of the wire. By means of this adjustment the wire can be pressed against the rubbing wheel, B, with any desired pressure. At the other end the wire, after passing between the jaws of a second vise, C, and over a pulley, supports a load which serves to hold the wire under tension. This second vise, weighing also 6 kg., is suspended by a two-point support,

so that although by its inertia it furnishes an effective end for the vibrating segment, it does not interfere with the slight longitudinal motion of the wire necessary to maintain a constant tension under varying conditions of temperature and adjustment. Although the tension has no direct effect on longitudinal waves, its exact value must be known for the computation of rubbing pressures from the settings of the feed screw.

Many different rubbing wheels were tried, some of which will be considered in detail later. They rotate about a vertical axis on centers which are mounted on the slide-rest of a small lathe-bed. This mounting makes possible the necessary adjustments of the wheels with respect to the wire. Power for rotation is furnished by a 1/8 H.P. induction motor, of which the speed under favorable conditions has been found to remain very nearly constant.

A transmission unit, consisting of a pair of counter shafts with cones of pulleys, reduces properly the motor speed, so that by shifting the belts the rubbing wheel can be given a peripheral speed having any one of thirty-three values between 6 and 50 cm. per second. Greater variety of speeds is obtained by changing the pulley of the motor shaft. An essential part of the transmission is a small fly-wheel mounted on centers and nicely balanced. This is placed close to the rubbing wheel and influences it through the single belt which furnishes power to both. All pulleys are of rolled brass, carefully turned and balanced. Endless braided linen belts, such as are used on dental engines, transmit the motions, and the design of the transmission prevents slipping and automatically regulates the tension of these belts. The constant positive motion of the rubbing wheel thus made possible has contributed much to the success of the experiments.

To examine the motion of a point on the wire a photographic method is used. A small plane mirror suitably mounted to allow free rotation about a fixed, vertical axis communicates with the wire and indicates by its angular position the wire's longitudinal displacement. The device is shown at *D* in Fig. 1. Light from a fixed "point-source," *E*, having passed through a suitable system of lenses, is reflected by this mirror and comes to a focus at a point on the surface of a photographic film wrapped around the cylinder, *F*. If the exposure be made while the cylinder rotates with uniform speed a record of the displacements of the wire plotted against *time* is obtained. A detailed description of the recording mechanism follows.

The small mirror is attached by beeswax to the flat side of a steel staff shown at *A* in Fig. 2. The staff is carefully fitted to rotate freely but without shake in cylindrical jeweled bearings with end-stones. A radial

arm, *B*, fits into a conical hole bored through the thick middle part of the staff. Two of these staffs were used, to each of which were fitted radial arms of various lengths. The larger, used for waves of greater

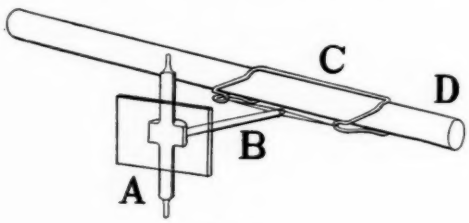

Fig. 2.

amplitude, measures 7 mm. in length and weighs 20 mg., while the smaller is but 4 mm. long and weighs 9 mg.[1]

The motions of the wire are communicated to the arm *B* through the spring clamp shown at *C*, and a fine silk thread. Such clamps are bent from natural hard music wire (no. 0000, diameter .215 mm.), are about 6 mm. long and weigh from 4 to 5 mg. each. Although they grip the vibrating string, *D*, with sufficient force to prevent slipping, their design permits them to loose their hold in case of accident before harm comes to the staff or jewels. Midway between the horns of the clamp the end of the staff's radial arm is held rigidly by fine silk thread.

This mechanism differs radically from those commonly used for similar purposes in that motions are produced in the staff positively and without the use of a "restoring force," usually derived from a spring. Springs, on account of their inertia, free period, and uncertain reactions, have proved to be a source of much trouble. The natural frequency of the new device is so great that no evidence of its existence has been found. The photographs of Helmholtzian waves agree with theory to a degree of approximation represented by 150 terms of a harmonic series; yet there appears no evidence of natural period so great as the smallest of these. Therefore the frequency must exceed 37,500 vibrations per second.

The mounting of the jeweled bearings proved to be a difficult problem. At first they were held rigidly in a fixed position, but the slight lateral motions of the wire, resulting from unavoidable irregularities in the rubbing surface, caused the staff to bind in its bearings. The resulting frictional losses interfered seriously with sub-Helmholtzian waves of slight stability. Worse than this was the danger to the staff from greater

[1] For the admirable workmanship on these pieces the writer is indebted to Mr. W. B. Hughes, in the employ of the B. C. Ames Co. of Waltham, Mass.

motions of the wire of an accidental nature. That the staff can endure rough usage is demonstrated by the fact that in responding to Helmholtzian waves its angular acceleration at the "corners" often exceeds 7,000,000 radians per second per second.

The mounting finally adopted is shown in Fig. 4. The bearings with the larger staff in place are shown at *A*. Together with necessary means of adjustment they are attached to the end of a steel rod supported near the middle in a universal joint, *B*, and balanced by a counterweight, *C*. This arrangement permits the staff to follow the wire in all of its transverse movements.

The bearings at *A* can be moved parallel to the wire through any desired distance by a turn of the micrometer head, *E*. Their movement produces a rotation of the staff equal to that caused by a longitudinal displacement of the wire through the same distance. In this way the film may be calibrated for magnification of the vibrations. The calibration, which requires a slight correction, is represented by the parallel straight lines in many of the photographs, for example, numbers *1*, *2* and *3* in Fig. 5. Magnifications of from 80 to 140 diameters were commonly used.

The entire unit shown in Fig. 4 rests upon a shelf which communicates with nothing except the brick wall to which it is attached. Since even this precaution did not suffice to prevent response to vibrations of the building, nearly all of the photographs were made at night.

A small self-feeding arc-lamp furnishes the necessary light. In front of it is mounted a screen of colored celluloid, upon which an image of the arc is formed by a lens. In order that holes of various sizes may be brought to the brightest part of the image, the screen is movable, an arrangement which provides "point-sources" of various sizes, circular in shape, evenly illuminated, and with sharp edges.

The cylindrical drum at *F*, Fig. 1, is shown again at *A* in Fig. 3, which is a vertical section of the photographic unit. The drum is of wood and measures about 15 cm. in diameter. It rotates on centers at speeds ranging from zero to 4,800 R.P.M. The corresponding velocities of the film vary from zero to 38 meters per second. Power is supplied by a small direct-current motor, direct-connected to the drum through a spring clutch.

[1] In many oscillographic experiments it is a difficult problem to get sufficient illumination. Granted a plane mirror, it is easy to show that the problem is solvable. Let it be assumed that the size of the mirror and the frequency and angular amplitude of its motion have been arbitrarily determined; also that in the photographic record is desired a certain "definition," or limiting ratio between the width of the line and its length per cycle. By adjustment of distances the amplitude in the record can be varied without changing the definition and the amount of "exposure" of the film varies inversely as the square of this amplitude.

The cylinder is enclosed in a light-tight box, *B*, provided with a simple hand-operated shutter, *C*, in front and a "loading sleeve," *D*, of black cloth behind. A method of attaching the specially adapted films has been devised, whereby they can be changed easily in a few seconds, yet are held with sufficient firmness to prevent tearing, even at the highest speeds. On account of the simplicity of the shutter it is difficult to restrict the duration of exposure to a single revolution of the film, and as a result many of the records show more than one curve.

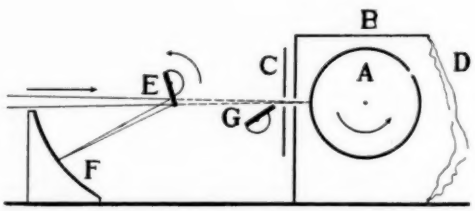

Fig. 3.

In front of the box and parallel to the axis of the cylinder, a shaft carrying a long narrow mirror, *E*, and a fly-wheel is mounted on centers and connected to the motor by a rubber belt. This mirror, by intercepting the beam of light on its way to the film and returning it to a curved screen, *F*, of white cardboard, makes possible visual observation of waves. The mirror is attached eccentrically to the shaft, which in turn is placed j st above the path of the light. The shaft has an angular speed of rotation about one third as great as that of the film. The mirror in its rotation intercepts the light for visual use during an interval corresponding to one revolution of the film, leaving it free for photographic use during the two following consecutive revolutions. By proper government of speed successive visual images may be made to coincide and to produce a persistent stationary curve which need not be interrupted during exposure of the film. Transient phenomena are thus easily studied. It should be noted, however, that no attempt is made to superpose images on the film.

This entire unit is mounted on leveling screws, which rest on the table of a heavy adjustable iron stand. It is not connected to any of the other pieces of apparatus. The precaution is necessary to prevent transmission of the slight vibrations caused by the rotation of the drum.

If observations are to be accurate, the axis of rotation of the film must be strictly parallel to the motion of the vibrating spot of light. To secure this relation a straight-edge, *G*, of white celluloid is carried on a shaft mounted in bearings, which are attached to box, *B*, in front of and

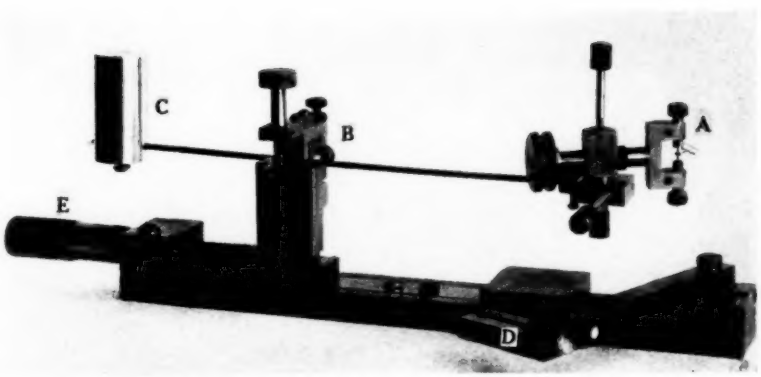

FIG. 4.

HARRY CLARK.

slightly below the level of the shutter. The parts are so adjusted that the edge in all of its possible angular positions remains parallel to the axis of the drum. When raised by hand the celluloid intercepts a small portion of the oscillating beam of light, and with adjustments well made the resulting illumination of the edge should be uniform.

III. Qualitative Description of Observed Phenomena.

It is proposed to devote this section to a qualitative presentation of longitudinal wave phenomena as observed with the aid of the apparatus just described, and to a brief discussion of some of the factors which enter into the successful production of these waves. In what follows it is to be assumed that both the rubbing wheel and the observing apparatus are located at the middle point of the wire.

In the earlier part of the work solids only were employed as friction agents, among them the various resins used on stringed instruments; also beeswax, and several mixtures. The surfaces of the wheels on which these agents were used were of various leathers, and of all thicknesses from two tenths of a millimeter to two millimeters. They varied greatly in quality. One was extremely hard, having been turned from discs of leather steeped in molten resin and tightly clamped together while hot. Another was extremely soft, consisting of a ring of thin chamois skin stretched over a wheel faced with thick gum-rubber. None of these devices, however, gave consistent results. Occasionally good sub-Helmholtzian waves were produced, but they could seldom be maintained long, and could not be duplicated at will. Although Helmholtzian forms could be produced, they often exhibited peculiarities of which something will be said later.

The influence of temperature was noticeable. Somewhat better results could be obtained at high temperatures than at low ones. When, in a cold room, a flame was applied to the wheel operating lightly with resin, the waves were appreciably modified. A considerable sharpening usually occurred, though this at times gave way to distortions.

The desired end was attained by the use of alcohol on a silk-covered wheel. This wheel, which was of brass, measured 42.2 cm. in diameter, and in its face was cut a shallow groove which served to guide the wire. Different thicknesses of silk were tried, and the best results were obtained by using perhaps ten layers of thinnest taffeta ribbon with a total thickness of less than a millimeter. The irregularity in the surface, caused by the free end of the ribbon, soon disappeared with use; the surface became hard and polished, and the accuracy of wave-forms increased. It may be of interest to note that this wheel, when nearly dry, gave the waves characteristic of resin.

Light pressure of the wire against the wheel, well saturated with alcohol, built up a wave which produced after a few minutes an audible tone of great purity. Continuing to grow, it assumed a permanent sub-Helmholtzian form, characterized by rounded "corners" and smooth symmetrical curves, and could be maintained indefinitely. Little could be learned about minimum building-up pressures. At low pressures the rate of building up was at first very small, and the necessary time varied, under fixed conditions, from twenty seconds to as many minutes.

As the pressure was increased, the wave gradually assumed the Helmholtzian form, the resulting tone closely resembling that of the violoncello. The corners of these waves, as shown by photographs, are surprisingly sharp. From the photographs it was estimated that the change of velocity is completed in less than .00001 second. When the pressure was then decreased, sub-Helmholtzian forms reappeared, which, by careful manipulation of pressure, could be carried down to very small amplitudes without loss of stability. In this way sub-Helmholtzian waves could be maintained, the amplitudes of which were not more than one-tenth or two-tenths as great as those built up directly from a state of rest under minimum pressure.

Some of the normal stable forms observed at the middle point are shown in Fig. 5, nos. *1*, *2* and *3*. These records, which are about one-half as large as the original photographs, were made at the same bowing speed and with equal magnifications. Nos. *2* and 3 are sine curves. This is true of all records of normal sub-Helmholtzian waves the amplitudes of which are not much greater than one-half that of the corresponding Helmholtzian. In Fig. 5 the time axis, in each case, extends from *left* to *right*, and the positive direction of the displacement axis, corresponding to the motion of the bow, is from *top* to *bottom* of the page.

It will be recalled that pressure was varied by a transverse movement of the vise at the end of the wire. The operation seemed to cause changes in the vibration system within, for it was accompanied by a marked increase in the dissipation of energy. To get the smallest amplitudes the pressure had to be decreased by small steps. It was moreover necessary to make each adjustment quickly and to allow time for the wave to recuperate before repeating the process.

All of these phenomena, described for a single bowing speed, were produced also at others. The forms of vibration suffered no change thereby, although the amplitudes of corresponding waves seemed to vary directly as the speed. At extremely low speeds it was found difficult to produce any waves whatsoever.

In addition to the experiments described above, a few trials were

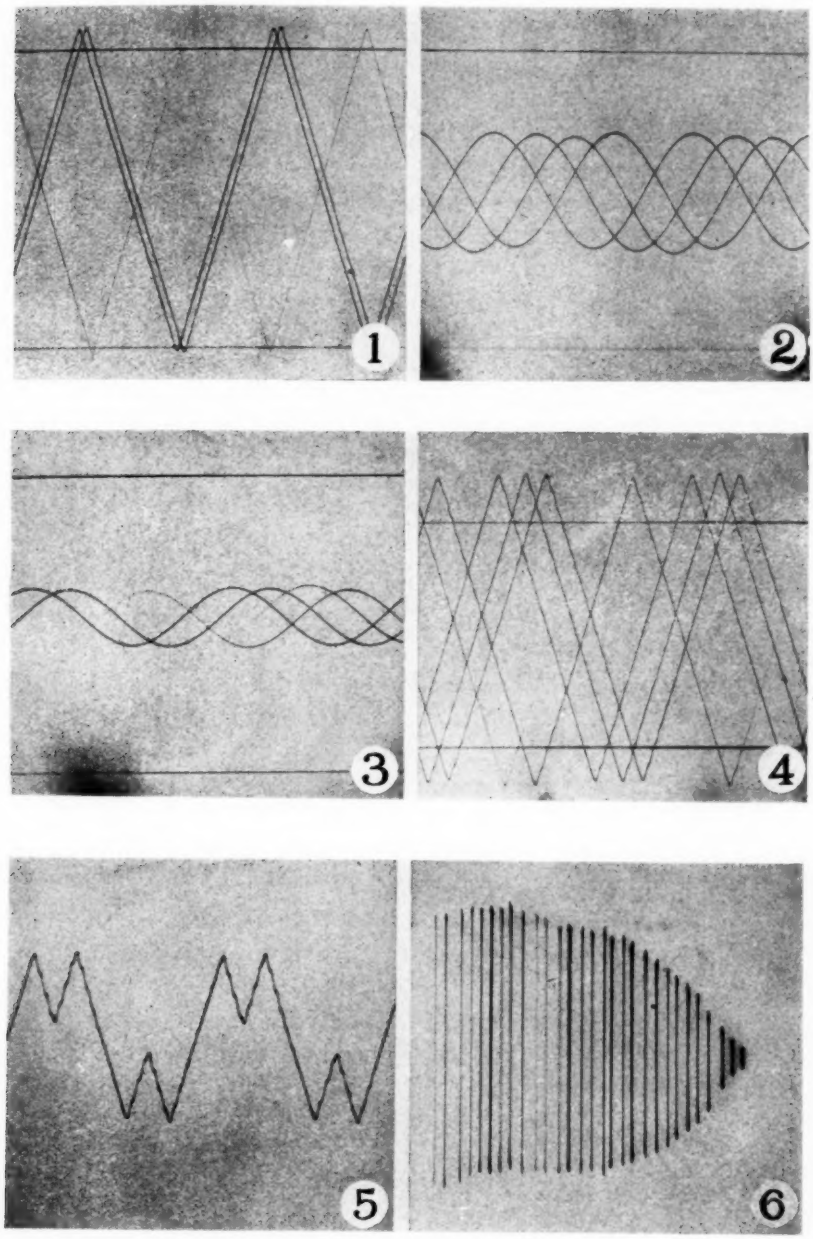

FIG. 5.

HARRY CLARK.

made in which the wire was excited by drawing the finger wet with alcohol along it. This stimulus produced rather sharp waves with sides curved in a characteristic fashion. When the finger was quickly withdrawn from the vibrating wire, the amplitude fell in about four seconds to one-half its original value. From this the rate of energy loss may be estimated, and it is found to be so small that even during rapid changes of wave form the wave equation is not violated.

It has already been seen that alcohol provides a far greater certainty of action than resin. It offers besides many other advantages. The wheel suitable to its use may be made almost truly circular, and its surface conditions, which are constant, may be readily duplicated. Unlike resin, alcohol operates with no observable temperature coefficient. Furthermore there are no incidental noises such as resin gives at the rubbed point. Finally, it is easier to handle. It is cleaner and requires less attention than resin.

Most of the conditions necessary for successful work have been pointed out in the description of apparatus. The question of tension in the wire remains. Although tension does not enter directly into simple longitudinal wave theory, it controls to a considerable extent the production of those transverse waves of slight amplitudes which have at times given trouble. Such transverse waves, it was found, could be prevented by the use of a rather high tension whose value was such as to give to the wire free transverse periods farthest removed from the period of the longitudinal wave. This tension was determined by experiment, and once learned did not need to be changed. Twenty-five kilograms gave good results.

IV. Quantitative Study of Sub-Helmholtzian Vibrations.

In this section will be presented the results of the experimental study of sub-Helmholtzian waves with special reference to the relations which connect speed, pressure, amplitude, and wave-form. For convenience the photographic records were studied by the following indirect method of procedure. The films were placed in a stereopticon, which projected their curves upon a screen of coördinate paper. When the screen had been adjusted to the proper distance and position, the curves were carefully traced with a pencil. In order to reduce them to the same degree of magnification, and also for other special purposes, all of the ordinates of the tracings had to be multiplied by the proper constant number and replotted. In much that follows it will be necessary to consider changes of the ordinates of curves whose abscissas remain unchanged. To avoid confusion, it will be found convenient to accept the following

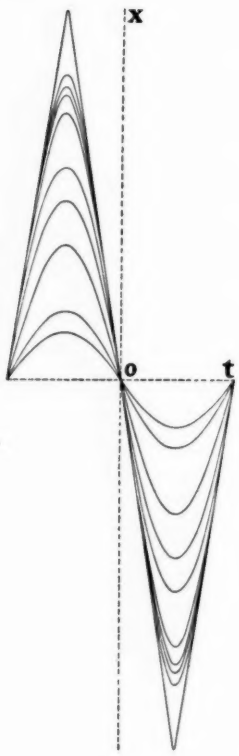

Fig. 6.

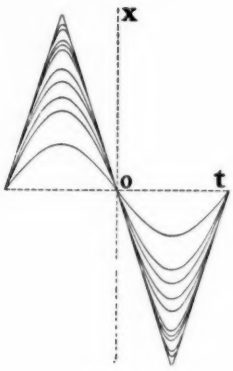

Fig. 7.

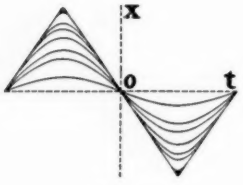

Fig. 8.

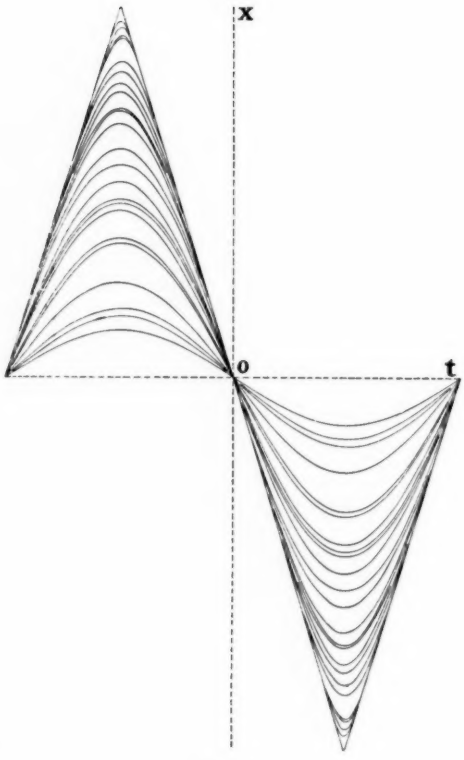

Fig. 9.

Definition.—Two curves are said to be *similar* if one of them may be produced by multiplying all of the ordinates of the other by a constant number. Figs. 6, 7 and 8 are reductions of curves treated as described above. Each of these figures represents the effect of different pressures at a single speed. The speeds, in cm. per sec., and corresponding

amplitudes, in centimeters, used in each case are given in the following table.

Figure.	Bow Speed.	Helm. Amplitude.
6	27.85	.0274
7	13.30	.0131
8	6.33	.00625

By careful measurements the limiting Helmholtzian amplitudes were determined and it was found, in verification of the law of Helmholtz, that their values are given by the ratio of *speed* to *four times the frequency*. Comparative studies of sub-Helmholtzian forms could not so readily be made, but some useful suggestions were obtained by a variation of the treatment of tracings, made clear by the following illustration: The ordinates of all of the curves of Fig. 7 were multiplied by a constant number whose value was so chosen that the resulting Helmholtzian curve in particular would be identical with that of Fig. 6. The curves of Fig. 8 were similarly treated with another proper constant. The three nests of curves were then superposed. All of the abscissas of the composite nest thus obtained were then multiplied by a constant, simply for convenience in illustration, with the result shown in Fig. 9.

A consideration of the figures above referred to, and of the methods by which they were produced, brings out several interesting facts. It is evident that *similar* curves may be produced at various speed by the application of suitable pressures. The waves are seen to be in a restricted sense symmetrical, since each positive displacement is followed, half a period later, by a negative one of equal magnitude. Symmetry about vertical axes is also indicated. Finally, it is evident that the amplitudes, not only of Helmholtzian waves, but also of the members of *any* family of *similar* waves, are directly proportional to the bowing speeds.

Since it had thus been shown that a normal sub-Helmholtzian wave is completely determined by its amplitude and bowing speed, certain simplifications of the work were possible. A slight variation of the photographic method, therefore, was introduced. In the original method all records were made on rapidly rotating films. The variation consisted in allowing the film to remain motionless to receive a very short exposure. The record, under these circumstances, is a narrow straight line, from the length of which the amplitude of vibration can be determined. The device offered the advantages of saving both time and material, for many exposures could be made upon a single film. Fig. 5, no. *6*, which illustrates the result, shows decrease of amplitude with decreasing pressure,

the bow speed having been kept constant. Under favorable conditions it was found possible to determine amplitude by direct measurement of the illuminated region of the screen without the aid of photography.

About six hundred determinations were made, some visually, others photographically, and the work covered a great variety of speeds and pressures. A single "run" consisted in determining the amplitudes corresponding at a single speed to a great many different pressures, and the results were embodied in a single plotted curve.

The manner of plotting requires some explanation. All of the amplitudes were modified by the same constant factor, the value of which was so determined that the modified Helmholtzian amplitude was equal to unity. All such curves representing the same speed should be identical. Most of the observed differences may be accounted for on the supposition that an inexact zero setting of the wire against the wheel introduced into all of the pressure readings errors constant for a given curve, but differing from curve to curve. Owing to the nature of the rubbing surface, and to the fact that the wire lies against it in a groove, it is difficult to determine the exact adjustment for zero pressure. An error of .01 cm. at this point introduces a pressure error of approximately one gram. The above explanation is therefore reasonable, inasmuch as the errors to be accounted for did not usually exceed one or two grams.

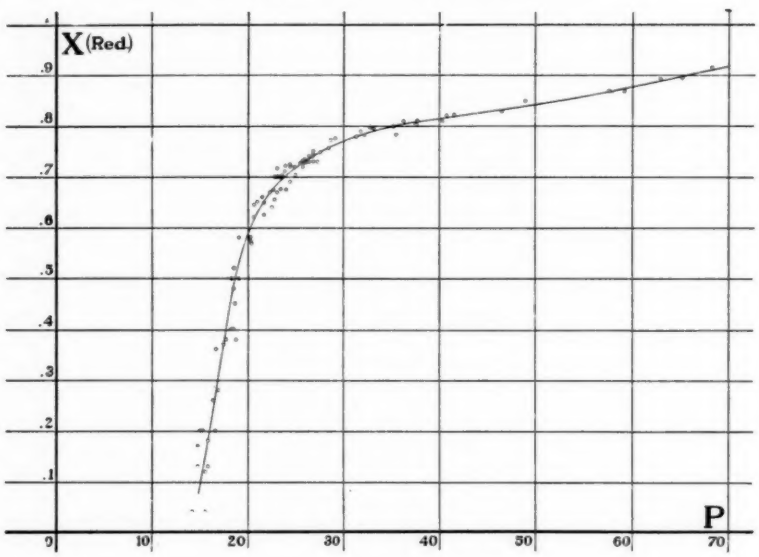

Fig. 10.

The method of averaging these amplitude-pressure curves may be briefly mentioned. They were superposed and if necessary displaced

along the pressure axis to give a best compromise curve. This new curve was then replotted with a proper displacement to represent the mean zero position. Such a curve, representing *all* of the determinations at one speed, is shown in Fig. 10. The distribution of points here shown

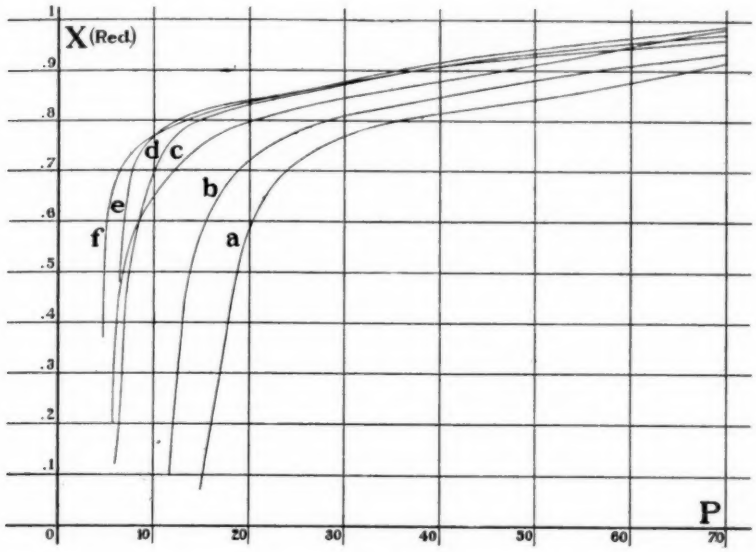

Fig. 11.

gives an estimate of the consistency of the data. Several of these compromise curves taken at widely different speeds are shown in Fig. 11, in which the pressures are given in grams. The following table gives the speed and corresponding Helmholtzian amplitude for each.

Curve.	Bow Speed.	Helm. Amplitude.
a	42.2	.0415
b	27.85	.0274
c	20.17	.0198
d	13.30	.0131
e	9.50	.00935
f	6.33	.00625

The curves are rendered comparable by the scale to which all of their ordinates have been reduced. Similar waves are represented in the figure by points having equal ordinates. At higher pressures, not here plotted, all the curves rise to within one or two per cent. of the limiting Helmholtzian. The speeds for *b*, *d* and *f* are the same as those illustrated in Figs. 6, 7 and 8.

Although the curves of Fig. 11 are very irregular, a few inferences may safely be drawn from them. They show that for the production of similar curves the necessary pressure increases with the speed. At speeds *a*, *b* and *c* the one is almost directly proportional to the other. Curves *d*, *e* and *f* on the other hand suggest a convergence of pressure upon a finite value as speed decreases indefinitely, and this apparent limiting value is far too great to be accounted for by errors in adjustment. In all of these curves, moreover, there is a certain similarity of shape which suggests similar laws of pressure and speed for all of the various wave-forms.

V. Minor Phenomena Peculiar to Helmholtzian Vibrations.

In connection with the experiments upon Helmholtzian waves it was found that any attempt to increase the sharpness of the corners by greatly increased pressure was usually accompanied by the production of ripples in the photographic curve. The phenomenon is shown in Fig. 5, no. *4*. Although these ripples are not in themselves particularly interesting, a consideration of them is rendered necessary by the possible inference that they do not exist in the wire's vibration, but instead are due to an imperfection of the observing apparatus. Their occurrence immediately after a sudden change in the direction of the string's motion points to a natural period of the oscillating mirror-system brought out by the very great acceleration which the system is at that instant undergoing.

It is not difficult to show, however, that this inference is untrue, and that the ripples constitute real phenomena of vibration in the string. In the first place their period could not be changed either by using staffs of different weights and dimensions, or by attaching considerable masses to the radial arms; and again, extremely sharp corners without ripples were sometimes observed. When resin was used ripples were very readily produced and were evident at times when the corners were far from sharp. On a few occasions they were seen superposed upon sinusoidal waves, which afterwards disappeared without destruction of the ripples. When very marked their pitch was easily recognized by the ear both before and after removal of the staff.

Their occasional occurrence in the straight parts of the Helmholtzian curve attests the innocence of the mirror, as does also the following consideration. A disturbance at any point of the wire is propagated in both directions and suffers reflection at the ends of the vibrating segment. The exact times of recurrence of these waves at its origin may be determined by an integral-surface construction. A photograph taken at $\frac{3}{4}$,

the bow being at $\frac{1}{4}$, not only exhibited ripples at the corners, but also showed their recurrence later at exactly the points indicated by theory. Ripples are therefore real vibrations of the string, caused doubtless by the bow, whose soft covering may have a highly damped natural period.

Besides ripples there were observed other unexpected phenomena peculiar to Helmholtzian waves. The following theoretical treatment will make clear their meaning.

It has been stated[1] that the wave-form is completely determined by the laws of Young, Helmholtz, and Krigar-Menzel. That this is not strictly true is made reasonable at least by a consideration of the following special case. With the bow at the middle point, let it be assumed that the whole string is divided into $(2n + 1)$ equal segments, n being a positive integer. The bow, being at the middle point of the middle segment may set up in this segment a complete Helmholtzian wave similar to the fundamental one and with amplitude and period equal to $[1/(2n + 1)]$ of those of the fundamental. If the same type of vibrations exist, with proper phase-difference, in all of the other segments, the wave equation, end conditions, and Young's law are satisfied, and the motion of the bowed part obeys perfectly the laws of Helmholtz. In other words these laws do not exclude the possibility of producing certain *Helmholtzian harmonics* as well as the fundamental Helmholtzian.

Since these laws show that the fundamental type of vibration is determined in all aliquot and rational cases by the position p/q of the bowed point, this fractional form may for present purposes be utilized to indicate the type of vibration even when applied to harmonics. For example, all of the harmonics described in the preceding paragraph may be said to be of the $(\frac{1}{2})$ type.

The impossibility of producing certain Helmholtzian harmonics is evident from two illustrations: first—the third harmonic, with the bow at $(1/3)$, violates Young's law; second—the second harmonic, with the bow at $(\frac{1}{4})$, violates the spirit if not the letter of Helmholtz's velocity law, because the type is so changed that the bowed point does not travel *with* the bow during so great a part of the entire period as the fundamental form would allow. The following statement defines completely all *possible* cases save those at $(\frac{1}{2})$. If the string be rubbed at any aliquot or rational point (p/q), where p and q are prime to each other, the production of the nth Helmholtzian harmonic is consistent with the laws of Young and Helmholtz if n and q are prime to each other. Its type of vibration is given by $(np/q - m)$ where m is a positive integer so chosen that

[1] Davis; see note 1.

$$0 < \left(\frac{np}{q} - m\right) < 1.$$

It is to be noted that although the wave equation and end conditions permit Helmholtzian harmonics to be superposed, the laws of the bow are thereby violated.

Special interest attaches to the $(\frac{1}{2})$ case, in which the motion of the middle point is so slightly restricted by the wave-equation and end conditions that it may be determined arbitrarily over an entire half-period. An infinity of complex Helmholtzian solutions exists therefore, and all necessary restrictions upon the forms of these solutions are defined by the following construction: Lay off on the time axis an interval of one half-period (of either fundamental or any odd harmonic), and connect its ends by any broken line which represents x as a single-valued function of t, and the segments of which are straight and have either one of two slopes corresponding to $\pm v$, where v is the bowing speed. When these forms are interpreted by means of an integral surface it is found that the displacement is zero along all sides of the square, so that all parts of the string pass through the position of equilibrium at the same instant. It has been pointed out by Rayleigh[1] that this is a necessary property of all natural free vibrations of strings. Since the periods of a string are not altered by the operation of rubbing,[2] the resulting vibration may be considered free and therefore, in one respect, the complex forms are *natural*.

The conditions for the production of all of these harmonics and other complex types are not as yet understood. Good rubbing wheels operating with alcohol commonly produced the simple "gable roof," which has always been considered the normal form. In contrast to this, however, hard resin wheels gave the more complex forms almost exclusively. These forms were apparently of infinite variety, and the vibration changed at times from one to another of them, slowly and without apparent cause. Complex forms could sometimes be produced with alcohol. To produce them it was necessary to remove the wheel along the wire to a distance of, say, three centimeters from the middle point. Once started, however, they usually persisted while the wheel was slowly drawn back to its normal position. History, then, as well as present conditions, determines type. In Fig. 5, no. 5, which shows a complex type produced with alcohol, the ripple is to be accounted for by the great bowing pressure required.

[1] Theory of Sound; Vol. 1, p. 180, 1894.

[2] F. Lippich; Mittheil. d. Deut. Math. Ges. in Prag., 1, 118, 1892.

VI. SUMMARY.

The results of the experiments described in this paper may be summarized as follows:

1. The ordinary Helmholtzian form at $(\frac{1}{2})$ has been photographically recorded with a degree of accuracy hitherto unapproached.

2. Helmholtzian forms have been discovered which demonstrate the inadequacy of the laws, as now stated, to determine completely the form of vibration. The limitations placed by present-known laws on the number and forms of these new vibrations have been determined.

3. The study of sub-Helmholtzian vibrations has not been restricted to the forms heretofore recognized, but has been extended to include those of very much smaller relative amplitudes.

4. It has been shown that under certain well determined conditions the tone produced by the rubbed string is of exceptional purity.

5. By improved methods, standard conditions have been determined sufficient for the production and duplication of normal waves, both Helmholtzian and sub-Helmholtzian.

A NOTE ON THE SPECTRUM OF RÖNTGEN RAYS.

BY O. J. ZOBEL.

IN an application of the Fourier analysis to radiation pulses which might result in giving the proper Röntgen spectrum, Sommerfeld[1] compares the relative merits of two types of pulses; the "one-sided" (electric force always of same sign during each pulse) and the "two-sided" (time integral of electric force equals zero for each pulse). Since in each case the electric vector is to be made up of a large number of these pulses occurring at random and without correlation as to sign during the time interval of expansion, the energy in any range of the spectrum per unit time will be the sum of the separate energies in the range arising from the pulses which occur during that time. The relative distribution of energy in the spectrum will then be the same as that resulting from a single typical pulse.

As a special case for the "one-sided" pulse Sommerfeld used

$$f(t) = \pm 1 \text{ for } 0 < t < \tau,$$
$$f(t) = 0 \text{ for } t < 0 \text{ and } t > \tau,$$

giving the spectral intensity

$$J_n = \left(\frac{2}{\pi n}\right)^2 \sin^2\left(\frac{n\tau}{2}\right),$$

where n is frequency.

For the "two-sided" pulse

$$f(t) = \pm 1 \text{ for } 0 < t < \tau,$$
$$f(t) = \mp 1 \text{ for } \tau < t < 2\tau,$$
$$f(t) = 0 \text{ for } t < 0 \text{ and } t > 2\tau,$$

there resulted

$$J_n = \left(\frac{4}{\pi n}\right)^2 \sin^4\left(\frac{n\tau}{2}\right).$$

From these formulæ Sommerfeld drew the following conclusion on page 725, line 17. "The 'one-sided' pulse has its maximum intensity at the frequency $n = 0$, the 'two-sided' has here the intensity zero and

[1] A. Sommerfeld, Ann. der Phys., 46, pp. 721–748. "Über das Spectrum der Röntgenstrahlung."

its maximum at $n = \pi/\tau$ approximately." Further on; "We see then that only the 'two-sided' pulse gives a spectrum, whose principal energy (for sufficiently small τ) lies in the extreme ultraviolet, and which therefore can serve to give the Röntgen spectrum. The 'one-sided' pulse shows a very improbable spectrum, since its principal energy lies in the most extreme infra-red." Extending his reasoning to any type of pulse he concludes on page 726, line 17, "Every 'one-sided' pulse, therefore, has its spectral maximum in the longest wave-lengths. On the other hand the intensity of the completely 'two-sided' pulse is here always equal to zero." And in line 24, "From this it appears that only a 'two-sided' pulse can produce an actual Röntgen spectrum."

Now the elimination of the "one-sided" pulse as the source of the Röntgen spectrum does not seem to me to be warranted on the above basis of reasoning as it can be shown that the conclusions drawn in regard to this type of pulse are unjustifiable. The discrepancy arises from the fact that the spectral intensity J when expressed in terms of frequency has its maxima at points which are different from the maxima points when J is expressed in terms of wave-length. For since

$$J_n dn = J_\lambda d\lambda$$

and

$$dn = \frac{2\pi C}{\lambda^2} d\lambda,$$

where C is the velocity of light,

$$J_\lambda = \frac{2\pi C}{\lambda^2} J_n.$$

Using the same special types of pulses employed by Sommerfeld we obtain for the "one-sided" pulse

$$J_\lambda = \frac{2}{\pi^3 C} \sin^2 \frac{\pi C \tau}{\lambda}$$

where J_λ has its broadest maximum at

$$\lambda_m = 2C\tau = C \cdot (\text{twice the duration of the pulse}).$$

(See curve 1.)

For the "two-sided" pulse

$$J_\lambda = \frac{8}{\pi^3 C} \sin^4 \left(\frac{\pi C \tau}{\lambda} \right),$$

with the broadest maximum at

$$\lambda_m = 2C\tau = C \cdot (\text{duration of the pulse}).$$

(See curve 2.)

This shows that both types of pulses give the broadest maximum in the spectrum at exactly the same wave-lengths, and that the other maxima occur at the same wave-lengths also. For the long wave-lengths the intensities vary as $(\lambda_m/\lambda)^2$ and $(\lambda_m/\lambda)^4$, which means but comparatively small energy in these regions for either type of pulse, the "two-sided" giving the least.

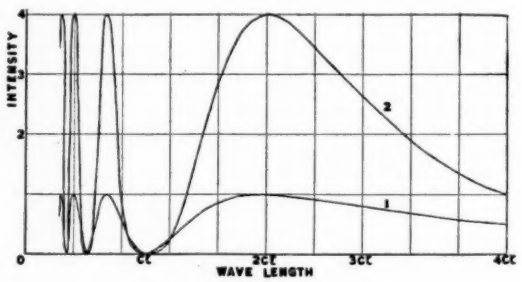

Fig. 1.

Thus the "one-sided" pulse does not have its broadest maximum (and other maxima) in the very long wave-lengths any more than does the "two-sided" pulse. Sommerfeld's objection to the former pulse has therefore lost its validity.

It is to be noted also that types of pulses other than these special ones used by Sommerfeld do not necessarily give more than a single maximum in the resulting spectrum.

PHYSICAL LABORATORY,
THE UNIVERSITY OF MINNESOTA,
February 10, 1916.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A NEW MERCURY VAPOR PUMP.[1]

BY HORATIO B. WILLIAMS.

THERE appeared last year in the Annalen der Physik a paper by Gaede in which was described a new high vacuum pump whose action depends on the difference in the rates of effusion of mercury vapor and gases through a very narrow slit. The writer constructed several of these pumps and a study of their behavior has led to the construction of a very simple pump which is much more rapid in action than the Gaede type and seems to be equally effective in producing extremely low pressures.

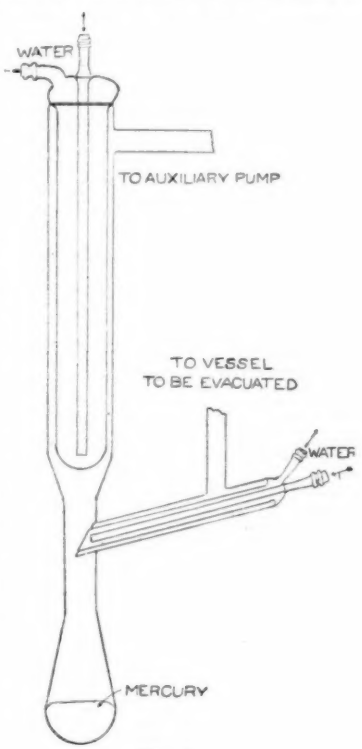

Fig. 1.

The vapor passes through a constriction connected to the vessel to be exhausted. Within the connecting side tube is a small Hopkins condenser (see accompanying sketch), whose cooling effect aided by that of the external air condenses any vapor which enters before it has penetrated far. Diffusion takes place through the annular space between the condenser and outer side tube. This space is 2 mm. wide and has an area of about one sq. cm. A larger Hopkins condenser above condenses the main stream of vapor. The auxiliary pump is connected to it near the top.

By arranging to have the diffusion take place at a constriction in the main vapor flow tube, advantage is taken of the Bernouilli effect. It is thus made possible to have the vapor pass the diffusion point rapidly without reaching a lateral pressure so high as to cause it to pass far into the side tube. A lip

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 26, 1916.

projecting from the side tube into the main flow tube hinders the direct passage of vapor into the side tube. The eddies in the vapor stream which this obstruction produces probably hasten diffusion. As the gas which diffuses into the stream of vapor is rapidly carried to the upper condensation chamber, the usually slow process of diffusion becomes very rapid. The auxiliary pump should produce a vacuum of something like .1 mm. of mercury; but the pump will operate with a fore vacuum of .25 mm. Its action is then much slower. Two pumps in series operate rapidly with a fore vacuum of .25 mm. The mercury may be heated with a small open flame, or preferably with a small electric oven. Effective action does not depend so critically upon precise regulation of temperature as in the Gaede type. A thermometer built in is unnecessary.

Preliminary experiments indicate that the exhaustion is even more rapid than with the molecular pump. A single pump with a May-Nelson oscillating ring pump as auxiliary exhausted a two-liter bulb with large aluminium electrodes from atmospheric pressure to a point where the alternative spark gap measured 18 cm. in 7 minutes. Two vapor pumps connected in series and sealed directly to a MacLeod gauge reduced the pressure from approximately .25 mm. to 4×10^{-5} mm. in 4 minutes. A short time later the pressure was too low to be measured.

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A RESONANCE METHOD FOR MEASURING THE PHASE DIFFERENCE OF CONDENSERS.[1]

BY H. L. DODGE.

IT is a well-known fact that when an alternating electromotive force is impressed upon a condenser the resulting current does not lead the electromotive force by the theoretical ninety degrees. This is particularly noticeable in the case of paper condensers of the telephone type so often used in laboratories for experiments in connection with the study of the alternating current circuit. When an attempt is made to interpret such experiments by graphic methods it is sometimes found that the power-factor of the condensers is so great that it must be taken into account. The angle by which the current lags behind the theoretical ninety degree lead is known as the phase difference of the condenser.[2] It is possible to determine this angle from data obtained in the experimental study of current resonance.

A coil of inductance L, resistance r, and impedance z is arranged to be connected in parallel with a number of paper condensers of nominally equal

[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 28–30, 1915.

[2] F. W. Grover, Bureau of Standards, Bull., Vol. 7, 495, 1911; Vol. 3, 371, 1907. These papers contain numerous references to other works upon this subject.

capacity. An alternating electromotive force being applied the values of the total current, the coil current, and the condenser current are read as the condensers are introduced one by one. The coil current will remain constant, the condenser current will increase uniformly, while the total current will decrease, pass through a minimum value and then increase.

If one chooses several sets of these values and constructs the vector diagram of currents he will find that if the phase difference of the condensers is to be zero the minimum total current must be exactly equal to the projection of the coil current upon the voltage axis. As a matter of fact the minimum total current will be found to be larger by an amount which varies with the magnitude of the phase difference.

To obtain the phase difference proceed as follows: Draw the voltage axis. From the constants of the coil determine the angle by which the coil current lags behind the voltage and plot to scale the vector representing the coil current. With the origin as a center and a radius equal to the minimum value of the total current describe an arc reaching a short distance each side of the axis of reference. From the extremity of the coil current vector draw a line tangent to this arc. The angle which this line makes with the reference axis is the power-factor angle of the condensers. The difference between this angle and ninety degrees is the phase difference of the condensers. It is also expressed by the formula

$$\text{Phase difference} = \cos^{-1}\frac{r}{z} - \cos^{-1}\frac{I_L}{I_T},$$

in which r and z are constants of the coil, I_L the coil current and I_T the minimum value of the total current. This expression is easily formulated from the vector diagram.

A sine-wave electromotive force has been assumed. If there are higher harmonics their effect will be to increase the apparent value of the phase difference. In the ordinary laboratory experiment this is not a serious matter unless the harmonics are very prominent. The effect of the harmonics may be reduced by the introduction of inductance in series with the parallel circuit.

This method is not proposed as one possessing any great accuracy. It is, however, of some theoretical interest and has proven to be of great practical value in a laboratory where students in studying the alternating current circuit use the commercial paper condenser.

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A SYNCHRONO-PHOSPHOROSCOPE.[1]

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IN the study of the phosphorescence of substances the glow of which is of brief duration, particularly where it is desired to investigate the changes of intensity or color due to heating or cooling, it is convenient to be able to deal with substances of considerable size. For this purpose the authors have devised and constructed the apparatus described in this paper.

The object to be examined is viewed through the open sectors of a revolving disk, *WW*, which is mounted on the shaft of a synchronous alternating current motor, *A.C.* The number of open, and of closed sectors equals the number of poles of the alternator and the excitation of the object being examined is by means of a spark or series of sparks *E*, or of a suitable vacuum tube, actuated by means of a step-up transformer *T.T.* on the same alternating circuit. By proper adjustment of the disk, excitation occurs during the intervals when the phosphorescent surface is behind the closed sectors and the surface is visible for successive intermediate intervals. On a 60-cycle circuit with four sectors these intervals are of 1/240 of a second each and may begin one thousandth of a second or less after the excitation ceases. By breaking the sparking circuit the subsequent decay of materials of long duration may be observed without stopping the disk.

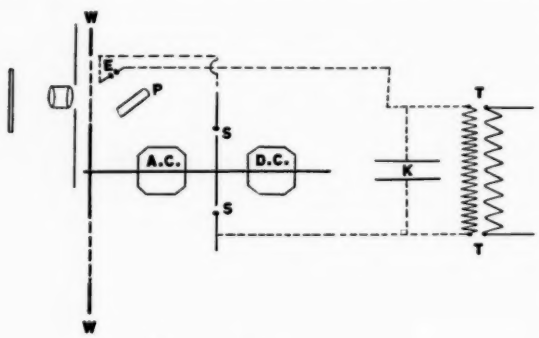

Fig. 1.

By means of a direct current motor, *D.C.*, on the same shaft, which ordinarily serves to bring the synchronous motor to speed, the disk may be driven at other speeds, the excitation in these cases being timed by means of a four point star wheel *SS* on the shaft so as to occur regularly during the passage of the closed sectors.

By the use of this device the authors have found it possible to take photographs in color by the Lumière process showing the change of color of various phosphorescent sulphides during decay and the effect of low temperatures on the color and intensity of these substances.[1] They have also made detailed comparisons of the spectra of the uranyl salts during and after the close of excitation and have determined the rate of decay of the phosphorescence of these salts.[2]

CORNELL UNIVERSITY,
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[1] Abstract of a paper presented at the Washington Meeting of the Physical Society, April 20–21, 1915.

[2] See Transactions of Am. Philos. Soc., 1916.

[3] See Proceedings of Nat. Acad. of Sci., 1916.